Efficient approaches for integrated aircraft scheduling and trajectory optimization at a busy terminal manoeuvring area

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ABSTRACT

This paper deals with the real-time problem of efficiently scheduling take-off and landing operations at a busy Terminal Manoeuvring Area (TMA). This problem is particularly challenging, since the TMAs are becoming saturated due to the continuous growth of traffic demand and the limited available infrastructure capacity. The mathematical formulation of the problem requires taking into account several features simultaneously: the trajectory of each aircraft should be accurately predicted, the safety rules between consecutive aircraft need to be modelled with high precision, the aircraft timing and ordering decisions have to be taken in a short time by optimizing performance indicators of practical interest, including the minimization of aircraft delays, travel times and fuel consumption. This work presents alternative approaches to integrate the various modeling features and to optimize the various performance indicators. The approaches are based on the resolution of mixed-integer linear programs via dedicated solvers. Computational experiments are performed on real-world data from Milano Malpensa in case of disturbed traffic conditions. The results obtained for the proposed approaches show different compromised solutions when prioritizing different indicators.

Keywords: Air Traffic; Take-off and Landing Operations; Optimal Control; Job Shop Scheduling; Mixed-Integer Linear Programming.
1 Introduction

As a result of the increase in air transport demand and traffic volumes of the last decades, airports have become the major bottlenecks in air traffic control. Due to the limited possibility to invest in airport construction or expansion, aviation authorities are exploring new ways to optimize the use of the existing infrastructure and to better manage landing and take-off aircraft movements in the vicinity of airports [16, 18, 39]. However, air traffic controllers have still limited support from automated systems [12, 24]. This is due to a lack of granularity in the management of aircraft trajectories, and to simplified aircraft scheduling techniques. A typical scheduling procedure consists of first computing a First Come First Served (FCFS) order of aircraft on the runways, and then improving it through small ad-hoc adjustments. The resulting schedules do not take into consideration the effects of the local aircraft timing and ordering decisions in the other TMA resources and in the medium term to the other aircraft. We believe that more advanced automated support is required in order to quickly compute (near-)optimal schedules in terms of a number of performance indicators and with a more precise simulation of aircraft trajectories in the vicinity of airports. In this way, air traffic controllers should be supported to reduce their workload, to take more effective decisions and to simplify the process of implementing the suggested solutions.

The mathematical formulation of the real-time take-off and landing aircraft scheduling at a busy TMA requires taking into account modeling features related to individual and multiple aircraft simultaneously: (i) the trajectory of each aircraft should be accurately predicted and possibly optimized according to performance indicators related to the specific characteristics of the chosen trajectories, e.g. to minimize airline costs, delays and/or environmental impact [1, 6, 42]; (ii) the aircraft timing and ordering decisions between multiple aircraft need to be modelled with high precision, in order to satisfy the safety rules between consecutive aircraft, and have to optimize global performance indicators, e.g. related to aircraft delays, travel times and fuel consumption (burned). This work presents alternative approaches to integrate the various modeling features of the problem and to optimize various performance indicators of practical interest.

A sub-problem relates to features (i) and is named Aircraft Trajectory Optimization Problem (ATOP) [22, 28, 31, 32]. ATOP is an optimal control problem in which a landing trajectory for each aircraft is computed by considering either the minimization of the travel time or the minimization of the fuel consumption. The aim is to compute the best free-flow trajectory for each aircraft in the vicinity of an airport in order to ensure the safety and efficiency of the flight during the landing procedure.

A detailed approach for planning the flying time of each aircraft should take into account the behaviour of all aircraft that share the same TMA resources. This information is required to identify a feasible schedule for all landing and take-off aircraft that is compliant with the safety regulations. Additionally, a good quality schedule needs to be selected among those that are feasible, and the schedule quality depends on the relevant economic and environmental performance indicators. This is the sub-problem related to features (ii) that is named Aircraft Scheduling Problem in a Terminal Manoeuvring Area (ASP-TMA) [9, 20, 10, 36, 37]. In particular, we study an ASP-TMA problem with heterogeneous aircraft and mixed runways.

The ATOP and the ASP-TMA are usually studied separately in the literature, see e.g. the reviews in [1, 6, 8, 29]. However, the optimal trajectories for multiple landing aircraft,
their sequencing and separation distance in a traffic optimization horizon should be determined simultaneously in the vicinity of an airport, taking into account the limitations due to multiple aircraft in the terminal airspace and minimizing the energy consumed, the travel time or the delays of the involved aircraft. Therefore, the two problems are two strongly interrelated sub-problems that need to be integrated in order to compute a feasible aircraft schedule for take-off and landing operations. This paper proposes alternative approaches to integrate the two sub-problems by taking into account single or multiple performance indicators. Each approach computes a compromised solution to the overall problem in a short time.

We model the ATOP as a mathematical formulation that incorporates the equations of motion, the physical and safety constraints imposed on the aircraft’s dynamics, and boundary conditions specific for each resource. The objective function is a linear combination of the minimization of traversal time and fuel consumption. For each aircraft traversing a set of TMA resources, a (near-)optimal solution is computed by OCPID-DAE1, a dedicated software developed at the Munich University of the Federal Armed Forces.

The ASP-TMA is modeled as a Mixed-Integer Linear Programming (MILP) formulation, that is based on a generalized job shop scheduling model with blocking (no-store), no-wait and additional constraints. This modeling approach is also known as alternative graph [26]. This formulation takes into account the airspace interactions between aircraft, the related TMA safety aspects and various performance indicators with a high level of detail. When dealing with the aircraft delay minimization, we solve the ASP-TMA via the algorithms implemented in AGLIBRARY, a dedicated software developed at Roma Tre University. When dealing with the travel time or fuel minimization, we use the commercial solver CPLEX.

This work presents various optimization-based approaches to solve the integrated problem. We first look at approaches that consider a single performance indicator. For all these approaches, we first solve ATOP and then solve ASP-TMA. In case the objective function is the aircraft delay or total travel time minimization, we first search for the minimum and maximum traversing times required by each landing aircraft in each resource, and then search for the optimal ASP-TMA with constraints on the minimum and maximum possible values of the traversing times. In case the objective function is the landing fuel consumption (travel time) minimization, we search for a solution to the ASP-TMA in which each landing aircraft flights in each resource of the TMA with its maximum (minimum) traversing time. The latter process can be viewed as a constraint satisfaction problem.

In order to take multiple performance indicators into account at the same time during the optimization process, we develop the following alternative approaches to integrate the ATOP and ASP-TMA solvers:

- **Lexicographic**: This approach first solves the ASP-TMA with aircraft delay minimization. The ATOP is then solved by minimizing either the fuel burned or the travel time, and without changing the aircraft ordering decisions of the ASP-TMA solution. As a result of this optimization process, the ASP-TMA and ATOP objective functions are optimized in a lexicographic order of importance.

- **Hybrid**: This approach solves the ASP-TMA formulation, with either delay or travel time minimization, by fixing the traversing time of each landing aircraft into each
TMA resource equal to either its maximum or minimum value, as computed by the ATOP solver. Compared to the constraint satisfaction problem on the landing fuel consumption (travel time) minimization, we here search for the best solution of the ASP-TMA problem, in terms of a given objective function, among the aircraft landing solutions with minimum landing fuel consumption (travel time).

- **Combined**: This approach first solves the ASP-TMA formulation with a single performance indicator, either delay or travel time minimization. Then, another ASP-TMA formulation is generated in which the optimal value of the previously optimized indicator appears as an additional constraint and the objective function is related to a different indicator. The optimal solution of the combined approach is optimal for the former objective and slightly far from optimum for the latter objective.

We did not consider approaches based on weighted objectives, since it would be very difficult to identify the right value of the weights, as shown in Samà et al. [35] for the combination of two objective functions. To summarize, the main contributions of this paper are: the integration of ATOP and ASP-TMA formulation in order to solve the aircraft take-off and landing scheduling problem, and the development of integrated approaches to solve the problem with consideration of multiple performance indicators.

Computational results are performed on practical-size instances of the Milano Malpensa (MXP) TMA in case of disturbed traffic conditions with multiple delayed aircraft. We compare the optimization approaches based on single performance indicators with the ones based on multiple performance indicators. We investigate the extent to which the latter approaches are able to find better compromise solutions compared to the former approaches in terms of various performance indicators.

The paper is organized as follows: Section 2 presents a review of the recent literature on the aircraft scheduling problem in the vicinity of airports; Section 3 formalizes the studied problem in terms of the various constraints and performance indicators; Section 4 briefly recalls the formulations used in this paper for the ATOP and the ASP-TMA; Section 5 and 6 introduce the solution approaches proposed for single and multiple indicators; Section 7 describes the computational results obtained on the MXP TMA and quantitatively compares the performance of the various approaches proposed; Section 8 discusses the paper findings and outlines further research directions. An appendix reports a detailed description and illustration of some numerical examples on the proposed formulations and solution approaches.

## 2 Literature review

This section presents a literature review on recent contributions to solve the aircraft landing and take-off scheduling problem. Our focus is on the problem modeling with specific attention on the choice of the objective functions and constraints. We organize the discussion based on the type of problem studied: aircraft arrival scheduling, aircraft departure schedule, and mixed arrival-departure scheduling.

Ball et al. [6], Allahverdi et al. [1], Bennell et al. [8] and Pellegrini and Rodriguez [29] present a detailed panoramic of different solving approaches and objective functions used in the aircraft scheduling literature. As reported in Bennell et al. [8], the main objectives
of airport traffic controllers are minimizing the arrival/departure delay and minimizing the approach time of aircraft before landing, while the airlines also look at the minimization of the operating (fuel) costs. In general, the main objective functions discussed are thus the minimization of delays and costs, and the latter are usually calculated based on the deviation from the landing/take-off time in published timetables or the travel time taken by the landing and/or take-off aircraft to perform the required operations. A few approaches focus on performing the landing and take-off operations with attention on the environmental impact, such as limiting the fuel burned or the noise emitted by aircraft, even if minimizing environmental effects is often the airport government preference.

In the arrival scheduling literature, Ernst et al. [13] measure the cost associated with the deviation from the preferred aircraft landing time. An allowable time window is assigned to each aircraft in order to perform the landing operations as well as earliness and tardiness penalties. Beasley et al. [7] study an aircraft displacement problem and consider the cost of a solution adjustment procedure where, if an aircraft is further delayed with respect to an initial solution, an additional penalty has to be paid. Hu et al. [23] minimize the sum of the difference between the predicted and the allocated landing times of each aircraft. Saraf and Slater [38] searches for an optimal arrival aircraft sequence based on predicted aircraft trajectories and within a maximum number of position switches from the FCFS order. Soomer and Franx [41] combine cost functions declared by the airlines for each aircraft typology and rescale them, making the resulting solution fair. Artiouchine et al. [2] assign landing times to aircraft for a single runway by minimizing the use of holding patterns or by maximizing the minimum time elapsed between two consecutive landings. In their approach, landing times are assigned to aircraft with a mono-pattern, where all aircraft are identical and where a single holding pattern is only considered. Artiouchine et al. [3] introduce another abstract model where the routing and scheduling problem for landing operations is formulated through the $K$ King problem. According to the authors, this approach is a strong abstraction of landing operations. For example, STI constraints are not accurately modeled, as well as the range of aircraft dynamics. Eun et al. [14] try to limit the aircraft delays and the deviation from the estimated arrival time, by taking into account airline preferences. Sölveling et al. [40] include in the cost function the environmental impact, in terms of fuel and CO$_2$ emissions, when there are deviations from the nominal schedule. Fisch et al. [19] first optimize the trajectory of each aircraft and then determine an optimal landing sequence for the conflicting aircraft via a discrete nonlinear program. Faye [17] presents a dynamic constraint generation algorithm for an aircraft landing problem with two potentially conflicting objectives: landing aircraft as earlier as possible or landing aircraft as close as possible to their scheduled landing time.

Other works on the runway scheduling problems aim to maximize the use of airport capacity (i.e., the throughput). Bianco et al. [9] consider a detailed model in which the TMA is formulated as a no-wait job shop scheduling problem with release times and sequence-dependent setup times to model aircraft safety separations and minimize the maximum completion time. Atkin et al. [4] focus on the development of an automated advisory system to help the runway controller to increase the throughput of the departure runway and to reduce the aircraft delay without negatively affecting safety and other feasibility constraints on the aircraft reordering. A weighted sum of delay, penalty and cost factors is minimized. The various factors in the objective function have an ordered degree of importance. Balakrishnan and Chandran [5] also adopted this objective function and discussed its practical importance in order to manage runway sequencing problems.
at major U.S.A. airports. Furthermore, they compare it with the minimization of the maximum aircraft delay and of the sum of aircraft delays.

The recent literature on the arrival and departure scheduling is next reported. Furini et al. [20] use pre-computed trajectories for the take-off and landing aircraft and propose an iterative rolling horizon algorithm and a tabu search heuristic in order to optimize the aircraft sequencing on the runway. Murca and Müller [27] developed a detailed arrival ordering and timing model with the integration of alternative landing trajectories from a set of standard terminal arrival procedures. Lieder and Stolletz [25] assume that landing times are assigned to the aircraft and search for an optimal aircraft sequencing on inter-dependent and heterogeneous runways, with minimization of a cumulative aircraft delay cost.

From the literature review, we highlight that recent research contributions are focusing on improving the level of precision in modeling the aircraft arrival and departure operations, introducing more real-world constraints and more accurate modeling of aircraft trajectories. An increasing stream of research is on modeling the problem as a job shop scheduling [9, 10, 11, 33, 34, 35, 36, 37], in which an operation denotes the traversal of a ground or air resource (air segment, common glide path, runway, holding circle) by a job (aircraft). In this paper, we use a MILP formulation that is a generalization of the Alternative Graph (AG) of Mascis and Pacciarelli [26], i.e. a special version of job shop scheduling model to deal with specific air traffic management constraints. The AG allows accurate modelling of relevant TMA aspects and safety constraints, such as waiting in flight before landing, flying in feasible time windows, hosting multiple aircraft simultaneously in air segments and single aircraft in runways. Specifically, the AG can model any 4-dimensional route for the aircraft in the TMA, while most of the related work done assumes 3-D routes are fixed and only optimizes the timing of runway operations.

The MILP formulation of AG can be efficiently solved by the rolling horizon framework of [37]. However, the latter approach requires a large computation time when dealing with complex ASP-TMA instances. For this reason, further research is dedicated to develop exact and heuristic algorithms for a single performance indicator optimization, i.e. the maximum delay minimization. D’Ariano et al. [10] develop a branch and bound algorithm for the problem of scheduling aircraft with fixed routes in the TMA, while other works [11, 33, 34] present advanced metaheuristics to solve an aircraft scheduling problem with flexible routes. Other extensions of the MILP formulation are proposed to deal with multiple performance indicators. Samà et al. [36] deal with problems related to aircraft delay minimization by taking into account aircraft priorities, violations and equity measures. Samà et al. [35] investigate the trade-off between aircraft delay and travel time minimization. This paper extends the latter approach in several directions: improving the level of detail in the management of landing aircraft trajectories, addressing the integration of single-aircraft trajectory optimization with multi-aircraft sequencing optimization, developing efficient approaches to compute trade-off solutions in terms of the following indicators: aircraft delay, travel time and fuel consumption.

3 Problem description

Let us consider a set of landing and take-off aircraft to be scheduled in the TMA during a pre-defined time period of traffic optimization. The minimum time at which an aircraft
can enter the TMA is its release time and depends on the aircraft position and speed at the start time \( t_0 \) of traffic optimization. A maximum entrance time, called deadline time, can also be considered for landing aircraft in order to model a time window of minimum and maximum entrance time in the TMA or even to fix the entrance time in the TMA equal to the release time. To complete its landing/take-off procedure, each aircraft is required to traverse a certain number of TMA resources (i.e., airborne and ground resources). The time required by an aircraft to traverse a resource represents the processing time on that resource.

An aircraft can be scheduled to start the processing of a specific TMA resource at a particular time. This scheduled time is called due date time. For all aircraft, we consider a due date time at the scheduled runway. For landing aircraft we call exit delay the delay of a landing aircraft at the runway with respect to its Scheduled Landing Time (SLT).

Take-off aircraft are supposed to depart within their assigned time window, and can be delayed in entering the TMA at ground level, before entering the runway. Following the procedure commonly adopted by air traffic controllers, we consider a time window for take-off between 5 min before and 10 min after the Scheduled Take-off Time (STT). A take-off aircraft has an exit delay if leaving the runway 10 min after its STT. In this paper, a ground delay does therefore not necessarily cause a delay at the runway.

A landing aircraft moves along a pre-defined route from an entry point in the TMA to a runway following a feasible descent trajectory. In case the TMA is congested, an holding pattern can be enforced by having the landing aircraft flying in circle in a holding circle, i.e. in an entrance buffer until the aircraft can be guided through its landing procedure in the TMA. Once entered a holding circle, the landing aircraft must travel at a fixed speed for at least half circle. After performing half circle of a loop the landing aircraft can reverse direction in order to continue the landing procedure. For each landing aircraft, there is a maximum number of allowed half circles, as prescribed by the air traffic controller. We assume that holding circle resources are uncapacitated. Landing aircraft then proceed toward the runway by traversing multiple air segments. The processing time in each air segment varies between a minimum and maximum possible value, according to the pre-defined maximum and minimum speed window available for each aircraft type.

The minimum and maximum processing time of each landing aircraft in each air segment is given by the solution of the ATOP, which accounts for the dynamics of movement of the landing aircraft in each traversed air segment. In this paper, the landing trajectories from the entrance in the TMA till the scheduled runway are optimized by looking either at the minimum travel time or at the minimum fuel consumption (in the latter case the travel time is maximized). When solving the ATOP, the characteristics of each individual aircraft and resource are first considered. Then, boundary conditions have to be imposed in order to assure feasibility of the aircraft’s trajectory through the different resources.

A minimum safety distance must be maintained between every pair of aircraft during all the approaching phases. This distance depends on the aircraft type and relative positions of the two aircraft (at the same or different altitude), and can be translated in a minimum separation time by considering the different aircraft types and their current speed and relative position. Minimum separation time is sequence-dependent, since the minimum distance between heavy, medium or light aircraft depends on the relative processing order of the common resources. Furthermore, two aircraft cannot overtake each other in an air segment, thus the entrance and exit aircraft order must correspond.
Similarly, a *runway* can be occupied by at most one aircraft at a time, and a minimum separation time should be ensured between any pair of aircraft.

A take-off aircraft leaves the runway moving towards its assigned exit point along an ascent profile, respecting safety separation standards. In case the TMA is congested, a take-off aircraft can wait on the ground before it is allowed to access the runway. In this paper, we do not control the ascent profile of take-off aircraft after leaving the runway, but we measure the possible take-off aircraft delay at the runway.

The ASP-TMA is the problem of assigning the start time of each operation, i.e. the traversing of a ground or air resource by an aircraft, such that all the potential conflicting situations between aircraft are solved, the safety constraints are respected and one or more performance indicators are optimized. This paper deals with three types of performance indicators: (i) the minimization of the maximum aircraft delay, (ii) of the total aircraft travel time, (iii) of the total energy consumed by the landing aircraft. Objective (i) results in a more compact schedule compared to other practical objectives, leaving more time available for accommodating future arrivals and departures. However, this objective usually involves the consideration of some strongly delayed aircraft. Objectives (ii) and (iii) present more global visions since they require the consideration of several aircraft. From the one hand, objective (ii) focuses on aircraft schedules in which each aircraft travels at its maximum speed in order to traverse the TMA as soon as possible. Specifically, we distinguish two cases: the minimization of the total travel time of all aircraft or of all landing aircraft during their landing procedure. On the other hand, objective (iii) deals with the fuel burned by each landing aircraft and requires traveling at the minimum speed in the TMA.

## 4 Problem formulation

This section describes briefly the mathematical formulations used in this paper in order to solve the ATOP and the ASP-TMA. Specifically, Section 4.1 presents the formulation developed by Palagachev et al. [28] and Rieck et al. [32], while Section 4.2 the formulation developed by Samà et al. [34, 37].

### 4.1 The ATOP formulation

The ATOP is the problem of computing the optimal trajectory of a landing aircraft in each TMA resource during its landing procedure. This problem disregards the presence of other aircraft in the TMA, and deals with either the minimization of the aircraft travel time or of the aircraft fuel consumption. The latter case can be viewed as the problem of maximizing the aircraft travel time.

Figure 1 illustrates two alternative landing trajectories for an aircraft flying in a resource $i$ of the MXP TMA: the trajectory represented by a solid line minimizes the travel time, while the trajectory represented by a dotted line minimizes the fuel consumption. In the former (latter) case, the travel time $\tau_i$ in resource $i$ is 120.5 (177.7) seconds.

The ATOP can be represented by a standard optimal control problem, in which a performance indicator is minimized (either the aircraft travel time or the aircraft fuel consumption), subject to feasible aircraft trajectory constraints; incorporating the equations of motion, the physical and safety constraints imposed on the aircraft’s dynamics,
and boundary conditions specific for each resource. In this paper, we consider a simplified, two-dimensional, point-mass model, which provides a good approximation of the real aircraft’s dynamics, while keeping the computational part of the framework simple enough [28, 32]. The aerodynamic data, the fuel flow data and the geometric data reflect the data included into the BADA data set [15].

Let us denote with \( z, h, V \) and \( \gamma \) the aircraft’s position, altitude, speed and climb angle respectively. We next refer to these variables as state variables. Standard procedure in optimal control theory is to assume that the state variables are functions in \( W^{1,\infty}([0, \tau_i], \mathbb{R}) \) (i.e. real-valued functions which are essentially bounded, together with their derivatives). We recall that \( \tau_i \) denotes the travel time in resource \( i \).

Let us also denote with \( T \) and \( \alpha \) the thrust and the control on the lift of the aircraft. These are commonly referred as control variables in optimal control theory, and generally assumed to be functions in \( L^\infty([0, \tau_i], \mathbb{R}) \) (i.e. real-valued functions which are essentially bounded). We can derive the lift (\( L \)) and drag (\( D \)) forces, acting on the aircraft at each instance \( t \in [0, \tau_i] \) by

\[
L(t) := \frac{\eta}{2} \cdot V(t)^2 \cdot S_{\text{ref}} \cdot C_L(t) \quad \text{and} \quad D(t) := \frac{\eta}{2} \cdot V(t)^2 \cdot S_{\text{ref}} \cdot C_D(t)
\]

where \( \eta \) is the air density, \( S_{\text{ref}} \) is the aircraft’s wing reference surface, while \( C_L \) and \( C_D \) are respectively the lift and drag coefficients, given by

\[
C_L(t) := C_{L_0} + C_{L_\alpha} \cdot \alpha(t) \quad \text{and} \quad C_D(t) := C_{D_0} + C_{D_2} \cdot C_L(t)^2.
\]

Note that \( C_{L_0}, C_{L_\alpha}, C_{D_0} \) and \( C_{D_2} \) are unitless parameters, depending on the aircraft type. The ATOP assumes the following form:

\[
\text{Minimize} \quad \omega \cdot \tau_i + (1 - \omega) \cdot \left[ \int_0^{\tau_i} T(t)/T_{\text{max}} \, dt \right]
\] (1)
\begin{align*}
\dot{z}(t) &= V(t) \cdot \cos \gamma(t) & \text{for a.e. } t \in (0, \tau_i) \quad (2) \\
\dot{h}(t) &= V(t) \cdot \sin \gamma(t) & \text{for a.e. } t \in (0, \tau_i) \quad (3) \\
\dot{V}(t) &= \frac{T(t) - D(t)}{m} - g \cdot \sin \gamma(t) & \text{for a.e. } t \in (0, \tau_i) \quad (4) \\
\dot{\gamma}(t) &= \frac{L(t) - m \cdot g \cdot \cos \gamma(t)}{m \cdot V(t)} & \text{for a.e. } t \in (0, \tau_i) \quad (5) \\
0 \leq T(t) &\leq T_{\text{max}} & \text{for a.e. } t \in (0, \tau_i) \quad (6) \\
0 \leq \alpha(t) &\leq \alpha_{\text{max}} & \text{for a.e. } t \in (0, \tau_i) \quad (7) \\
V_{\text{min}} \leq V(t) &\leq V_{\text{max}} & \text{for every } t \in (0, \tau_i) \quad (8) \\
LOAD_{\text{min}} &\leq \frac{L(t)}{m \cdot g} \leq LOAD_{\text{max}} & \text{for every } t \in (0, \tau_i) \quad (9) \\
V(t) \cdot \sin \gamma(t) &\leq 0 & \text{for every } t \in (0, \tau_i) \quad (10) \\
(z, h, V, \gamma)(0) &= (z_0, h_0, V_0, \gamma_0) & \quad (11) \\
(z, h, V, \gamma)(\tau_i) &= (z_i, h_i, V_i, \gamma_i) & \quad (12) \\
\tau_i &\in [0, +\infty) & \quad (13) \\
(z, h, V, \gamma) &\in W^{1,\infty}([0, \tau_i], \mathbb{R}^4) & \quad (14) \\
(T, \alpha) &\in L^{\infty}([0, \tau_i], \mathbb{R}^2) & \quad (15)
\end{align*}

The parameter \( \omega \) in Equation (1) is responsible for switching between the minimum travel time (\( \omega = 1 \)) and the minimum fuel consumption (\( \omega = 0 \)) problem. The integral term in (1) does not penalizes the real fuel consumption, but positive values of the control variable \( T \). The former has been adopted for numerical stability reasons, nevertheless it is not restrictive, as in general the fuel consumption is proportional to the applied \( T \). Equations (2),(3),(4) and (5) are related to a simple point mass model, including the states related to \( z, h, V \) and \( \gamma \). In Equations (4) and (5), we denote with \( m \) and \( g \) the aircraft mass and the gravitational acceleration respectively. Equations (6),(7) and (8) bound the values of \( T(t), \alpha(t) \) and \( V(t) \). In Equations (9) we limit the ratio \( L(t)/(m \cdot g) \), which is known as the load factor between \( LOAD_{\text{min}} \) and \( LOAD_{\text{max}} \), in order to ensure safe and comfortable trajectory. Equations (10) allow no ascending trajectories. Finally, Equations (11) and (12) fix the state variables at the initial and final time, and are strictly related to the resource traversed by the aircraft.

### 4.2 The ASP-TMA formulation

The ASP-TMA can be modeled as the alternative graph \( G = (N, F, A) \) of [26]. We let \( N = \{0, Bi, ..., Cj, n\} \) be the set of nodes, nodes 0 and \( n \) represent the start and the end of the schedule, while the remaining nodes \( \{Bi, ..., Cj\} \) are related to the start of the ASP-TMA operations. We denote \( Bi \ [Cj] \) as the operation related to the traversing of resource \( i \ [j] \) by an aircraft \( B \ [C] \). The start time \( t_{Bi \ [Cj]} \) of operation \( Bi \ [Cj] \) is the entrance time of aircraft \( B \ [C] \) in resource \( i \ [j] \). In particular, we fix the start time of the schedule at a known value, e.g. \( t_0 = 0 \), while the end time of the schedule is a variable \( t_n \).

\( F \) is the set of fixed directed arcs that model the processing, release, deadline and due date times related to the operations of each aircraft. Each fixed directed arc \( (Bi, Cj) \in \)
$F$ connects the two nodes (operations) $Bi$ and $Cj$. The weight $w_{Bi,Cj}^F$ of arc $(Bi, Cj)$ represents a minimum time constraint between $t_{Bi}$ and $t_{Cj}$ (i.e., $t_{Cj} - t_{Bi} \geq w_{Bi,Cj}^F$). To model the routing of aircraft $B$, we let $Br(i)$ denote the operation following $Bi$. The fixed arc $(Bi, Br(i))$ has a weight $w_{Bi,Br(i)}^F = w_{Br(i),Bi}^F$ equal to the minimum (− maximum) time required by aircraft $B$ to process resource $i$. The fixed arc $(0, Bf)$ models the release [deadline] time of operation $Bf$, i.e., the minimum [maximum] time at which aircraft $B$ can start processing the first resource $f$ of its route. Moreover, the fixed arc $(Br, t)$ with weight $w_{Br,t}^F$ models the due date time of operation $Br$, i.e., the maximum time at which aircraft $B$ can start processing the scheduled runway $r$ without incurring in a delay.

$A$ is the set of alternative pairs that model the aircraft sequencing and holding circle decisions. Each alternative pair is composed of two alternative directed arcs $((Bi, Ch), (Cj, Bk)) \in A$ that model either aircraft holding circle decisions (when $B = C$) or aircraft sequencing decisions (when $B \neq C$). In the former case, the alternative pair is rewritten as follows: $((Bi, Br(i)), (Br(i), Bi)) \in A$, in which $Bi [Br(i)]$ is related to the entrance in [exit from] the holding circle. This type of alternative pair models the number of half circles assigned to aircraft $B$ in the holding circle. In the latter case, the alternative pair $((Bi, Ch), (Cj, Bk)) \in A$ models the sequencing decisions at an air segment or at a runway.

In any ASP-TMA solution, only one arc of each pair in the set $A$ can be selected. For the alternative pair $((Bi, Br(i)), (Br(i), Bi)) \in A$, the two alternative arcs $(Bi, Br(i))$ and $(Br(i), Bi)$ model a different number of half circles assigned to aircraft $B$, and the weights $w_{Bi,Br(i)}^A$ and $w_{Br(i),Bi}^A$ model the different processing times in the holding circle. For the alternative pair $((Bi, Ch), (Cj, Bk)) \in A$, the weight $w_{Bi,Ch}^A [w_{Cj,Bk}^A]$ represents the minimum sequence-dependent separation time when aircraft $B$ proceeds $C$ [when aircraft $C$ proceeds $B$]. In general, if alternative arc $(Bi, Ch) [(Cj, Bk)]$ is selected in a solution, the constraint $t_{Ch} - t_{Bi} \geq w_{Bi,Ch}^A [t_{Bk} - t_{Cj} \geq w_{Cj,Bk}^A]$ has to be satisfied.

A selection $S$ is a set of alternative arcs obtained by selecting exactly one arc from each alternative pair in $A$ and such that the resulting graph $G(F, S) : (N, F \cup S)$ does not contain positive-weight cycles. A selection $S$ is a solution for the ATC-TMA problem. Given a selection $S$ and any two nodes $Bi$ and $Ch$, we let $l^S(Bi, Ch)$ be the weight of the longest path from $Bi$ to $Ch$ in $G(F, S)$. By definition, the start time $t_{Bi}$ of $Bi \in N$ is the quantity $l^S(0, Bi)$, which implies $t_0 = 0$ and $t_n = l^S(0, n)$.

The alternative graph of the ATC-TMA problem can be formulated as a disjunctive program [37]. This is achieved via a big-$M$ formulation in which there is an exact correspondence between arcs and constraints: each fixed directed arc translates into a fixed constraint, while each alternative pair into a pair of alternative constraints. We next present a compact big-$M$ formulation, while a detailed formulation is given in [34]. We recall that $t_0 = 0$ is the start time of traffic optimization. For each operation $Bi \in N$ there is a non-negative real variable $t_{Bi}$ modelling its start time. For each alternative pair $((Bi, Ch), (Cj, Bk)) \in A$ there is a binary variable $x_{Bi,Ch}^{Cj,Bk}$ modelling the sequencing/holding decision.

\[
\begin{align*}
\min & \quad f(t) \\
\text{s.t.} & \quad t_{Bi} - t_0 \geq w_{0,Bf}^F & \forall (0, Bf) \in F \\
& \quad t_0 - t_{Br} \geq w_{Bf,0}^F & \forall (Bf, 0) \in F \\
& \quad t_{Br} - t_0 \geq w_{Br,n}^F & \forall (Br, n) \in F
\end{align*}
\]
\[
t_B - t_B\sigma(i) - t_B \geq w_{B_i,B\sigma(i)}^F \quad \forall (B_i, B\sigma(i)) \in F
\]

\[
t_B - t_B\sigma(i) + Mx_{B_i,B\sigma(i)}^A \geq w_{B_i,B\sigma(i)}^F \quad \forall (B\sigma(i), B_i) \in F
\]

\[
t_B\sigma(i) - t_B + Mx_{B_i,B\sigma(i)}^B \geq w_{B_i,B\sigma(i)}^A \quad \forall ((B_i, B\sigma(i)), (B\sigma(i), B_i)) \in A
\]

\[
t_C - t_B + Mx_{B_i,B\sigma(i)}^B \geq w_{B_i,B\sigma(i)}^A \quad \forall ((B_i, B\sigma(i)), (B\sigma(i), B_i)) \in A
\]

\[
t_B - t_B\sigma(i) + M(1 - x_{B_i,B\sigma(i)}^B) \geq w_{B_i,B\sigma(i)}^A \quad \forall ((B_i, B\sigma(i)), (B\sigma(i), B_i)) \in A
\]

The objective function is in Equation (16). We next describe the ATC-TMA problem constraints.

Constraints (17) and (18) model the fixed directed arcs \((0, Bf)\) and \((Bf, 0)\) in \(F\), that represent the release and deadline time constraints related to the first of operation \(Bf\) of aircraft \(B\). Constraints (19) model the fixed directed arcs \((Br, n)\) in \(F\), that represent the due date time constraints related to the runway operation \(Br\) of aircraft \(B\).

Constraints (20) and (21) model the fixed directed arcs \((Bi, B\sigma(i))\) and \((B\sigma(i), Bi)\) in \(F\), that represent respectively the minimum and the – maximum processing times related to operation \(Bi\).

Constraints (22) model the alternative pairs \(((Bi, B\sigma(i)), (B\sigma(i), Bi)) \in A\). These alternative pairs model holding circle decisions regarding aircraft \(B\), thus each arc of the pair connects two operations of the same job. In any ATC-TMA solution, only one arc for each alternative pair must be active. The activation of one arc for this type of pair is modeled by the pair of constraints in (22), and corresponds to fixing the number of holding circles (if any) to be performed by each landing aircraft before the landing procedure.

Constraints (23) model the alternative pairs \(((Bi, Ch), (Cj, Bk)) \in A\). Each of these alternative pairs model the two possible sequencing decisions between a pair of aircraft \((B\) and \(C)\) at either a shared air segment or a shared runway. Indeed, the arcs of this type of alternative pair connect two operations of different jobs (aircraft). The activation of one arc for this type of pair is modeled by the pair of constraints in (23), and corresponds to enforcing a particular sequence between the two aircraft. In general, the alternative arc \((Bi, Ch)\) is active (i.e. enforces \(t_{Ch} - t_{Bi} \geq w_{Bi,Ch}^A\)) when \(x_{Bi,Ch}^C = 0\), while the alternative arc \((Cj, Bk)\) is active (i.e. enforces \(t_{Bk} - t_{Cj} \geq w_{Cj,Bk}^A\)) when \(x_{Bi,Ch}^C = 1\).

5 Integrated approaches with a single performance indicator

This section describes a first group of approaches to integrate the ATOP and ASP-TMA with optimization of a single performance indicator. All these approaches are based on a two-step method: we first solve the ATOP formulation of Section 4.1 for each landing aircraft by setting the parameter \(\omega = 1\) \((\omega = 0)\) in order to compute the minimum (maximum) travel time in each TMA resource traversed by the landing aircraft. We then
search for an optimal ASP-TMA solution by solving a formulation similar to the one of Section 4.2 in which the fixed constraints may vary depending on the minimum and/or maximum travel time constraints plus the release, deadline and (eventually) due date time constraints. We consider the following approaches:

**Maximum delay minimization** : This is an adaptation of the formulation of Section 4.1. Considering \( t_0 = 0 \), the objective function is set as follows: \( \min t_n \). A connected graph \( G(F,S) \) corresponds to an optimal solution if the selection \( S \) presents the minimal \( t_n = l^S(0,n) \) over all possible solutions. Minimum and maximum processing time constraints are given for each landing aircraft in each air segment. A minimum processing time is given at runways for each aircraft. Each landing/take-off aircraft has a release time constraint to force a minimum entrance time in the TMA. For each landing aircraft, a deadline time constraint fixes the entrance in the TMA equal to its release time. In this way, landing aircraft can only be delayed inside the TMA. All landing and take-off aircraft have a due date time constraint on their scheduled runway. The maximum delay minimization formulation is:

\[
\min t_n
\]

*Constraints (17), (18), (19), (20), (21), (22), (23), (24), (25), (26)*

**Total travel time minimization** : In this case, the total travel time of all aircraft traveling in the TMA is minimized. We thus have the following objective function: \( \min \sum_{B=1}^{\text{total aircraft}} (t_{Bl} - t_{Bf}) \), in which \( t_{Bf} \) \( (t_{Bl}) \) is the entrance (exit) time of aircraft \( B \) in (from) the first (last) resource \( f \) \( (l) \) of its route. The fixed and alternative constraints are the same as for the maximum delay minimization formulation except for the exclusion of the due date time constraints, since these do not affect the objective function. The total travel time minimization formulation is:

\[
\min \sum_{B=1}^{\text{total aircraft}} (t_{Bl} - t_{Bf})
\]

*Constraints (17), (18), (20), (21), (22), (23), (24), (25), (26)*

**Landing fuel burned minimization** : With this formulation, we search for a solution to the ASP-TMA in which each landing aircraft must traverse each air segment of the TMA with its maximum travel time. The latter process can be viewed as a constraint satisfaction problem in which we search for a feasible schedule that satisfies the following constraints for each landing aircraft \( B \) in each TMA resource \( i \):

\[
t_{Bi} - t_{B\sigma(i)} \geq w^F_{Bi,B\sigma(i)}; \ t_{Bi} - t_{B\sigma(i)} \geq w^F_{B\sigma(i)Bi} \text{ and } w^F_{Bi,B\sigma(i)} = -w^F_{B\sigma(i)Bi}.
\]

In this case \( w^F_{Bi,B\sigma(i)} \) is set equal to the maximum possible traversing time \( \delta_{Bi} \) of \( B \) in resource \( i \) as computed by ATOP. Due date time constraints are not included, since aircraft delays are not minimized. In any solution to this special ASP-TMA problem, all landing aircraft consume the less possible energy. The constraint satisfaction formulation related to the landing fuel burned minimization is:

\[
w^F_{Bi,B\sigma(i)} = w^F_{B\sigma(i)Bi} = \delta_{Bi} \quad \forall Bi \in N
\]

(27)
Landing travel time minimization: Here, each landing aircraft must traverse each resource of the TMA with its minimum travel time. This is formulated by modifying the fuel burned minimization formulation as follows: all fixed and alternative constraints are the same. However, for each landing aircraft $B$ in each resource $i$ the weights $w_{Bi,Be(i)}^F$ and $w_{Be(i),Bi}^F$ are fixed equal to its minimum traversing time $\rho_{Bi}$ as computed by ATOP. The resulting constraint satisfaction formulation is:

\[
C(17), (18), (20), (21), (22), (23), (24), (25), (26)
\]

\[
w_{Bi,Be(i)}^F = w_{Be(i),Bi}^F = \rho_{Bi} \quad \forall Bi \in N
\]

6 Integrated approaches with multiple performance indicators

This section presents a further group of approaches to integrate the ATOP and the ASP-TMA. The main difference with respect to the approaches of Section 5 is the consideration of multiple performance indicators during the optimization process. All the proposed methods are inspired on the fact that different optimal solutions often exist for a given performance indicator, and one can select an optimal solution such that a secondary performance indicator is also optimized. We develop integrated approaches based on the lexicographic, hybrid and combined methods introduced in Section 1.

We recall that the idea behind the lexicographic method is to solve the ASP-TMA and ATOP sequentially (i.e. the former problem is to compute the optimal aircraft sequencing for a primary performance indicator and the latter is to optimize a secondary indicator by varying the aircraft timing only). The other two methods are based on the resolution of MILP formulations that incorporate multiple indicators. This is achieved by minimizing a particular indicator and by taking into account other indicators as additional constraints. The hybrid method formulates the ASP-TMA with the addition of strong constraints on the minimum or maximum traversing time of landing aircraft in each air segment. These additional constraints are used to force the trajectories provided by ATOP. The combined method is based on the resolution of two ASP-TMA formulations with light constraints on the landing aircraft trajectories. The first formulation is to optimize a first performance indicator by disregarding the other performance indicator, while the second formulation is to optimized the second performance indicator by constraining the solution to be optimal for the first performance indicator (i.e. by adding a constraint on the optimal value of the first indicator).

The next subsections describe in more detail the integrated approaches with consideration of multiple performance indicators developed in this paper. We propose the lexicographic, hybrid and combined methods for a number of performance indicators described in Section 3. Specifically, we identify an integrated approach with multiple indicators based on the chosen method (lexicographic, hybrid or combined) and on the performance indicators considered as follows: Method <primary indicator, secondary indicator>.

Lexicographic <Delay, Landing Travel>: This is a two-step lexicographic optimization approach. We first compute an optimal solution for the ASP-TMA with maximum delay minimization, as formulated in Section 5. The resulting connected graph $G(F,S)$ describes the aircraft timing and sequencing of this solution. We
then re-optimize the timing of each landing aircraft in the ATOP by solving the formulation of Section 4.2 with minimum travel time ($\omega = 1$) and by constraining the travel time $\tau_i$ on resource $i$ between a minimum $\tau_i^{\min}$ and a maximum $\tau_i^{\max}$ value (Constraint 29) computed such that all the constraints in $F$ and $S$ are respected and $t_n$ in $G(F,S)$ is minimum [30].

$$\min \tau_i$$

Constraints (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13), (14), (15)

$$\tau_i^{\min} \leq \tau_i \leq \tau_i^{\max}$$

(29)

This approach has the advantage to compute an optimal solution for the maximum delay minimization in which the travel time of landing aircraft is minimized as a secondary indicator.

**Lexicographic <Delay, Landing Fuel>**: This is another two-step lexicographic optimization approach. The aim is to search for a solution with minimal fuel consumption among the optimal solutions for the ASP-TMA with maximum delay minimization. Compared to the other lexicographic approach, the aircraft sequencing is the same (i.e. as in $G(F,S)$), while the timing of each landing aircraft is re-optimized in the ATOP as follows: we solve the formulation of Section 4.2 with minimum fuel consumption ($\omega = 0$) and constrain the travel time $\tau_i$ on resource $i$ between $\tau_i^{\min}$ and $\tau_i^{\max}$, as formulated in (29).

$$\min \tau_i + \int_0^{\tau_i} T(t)/T_{max} dt$$

Constraints (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13), (14), (15), (29)

**Hybrid <Landing Travel, Delay>**: This approach is to solve an extended version of the ASP-TMA formulation for maximum delay minimization (secondary indicator) in which Constraint (28) is added in order to force the landing aircraft trajectories with minimum landing travel time (primary indicator). The advantage of this formulation is that landing aircraft spend exactly the least possible time on the landing air segments of the TMA. However, this hybridization may force some landing aircraft to travel half circles in the holding circle resources before approaching their landing procedure. This may generate a sub-optimal ASP-TMA solution in terms of maximum delay minimization.

$$\min t_n$$

Constraints (17), (18), (19), (20), (21), (22), (23), (24), (25), (26), (28)

**Hybrid <Landing Travel, Total Travel>**: This hybrid MILP formulation is characterized by the addition of Constraint (28) into the ASP-TMA formulation for total travel time minimization (secondary indicator). With this formulation, the travel time of both landing and take-off aircraft is minimized. However, the hybridization gives priority to landing operations since the landing aircraft must use the trajectories with minimum travel time (primary indicator), as enforced by Constraint (28).

$$\min \sum_{B=1}^{\text{total aircraft}} (t_{Bl} - t_{Bf})$$
Hybrid <Landing Fuel, Delay>: This approach is based on a hybrid MILP formulation that unifies the ATOP formulation for fuel consumption minimization (primary indicator) with the ASP-TMA formulation for maximum delay minimization (secondary indicator). The hybrid formulation is obtained by adding Constraint (27) in the original ASP-TMA formulation for maximum delay minimization. Any solution of this hybridization is optimal in terms of the minimization of landing fuel consumption. However, the optimal solution of this formulation may be sub-optimal in terms of maximum delay minimization, since landing aircraft are forced to traverse the air segments of the TMA with a larger travel time compared to their minimum possible travel time.

\[
\min t_n
\]

Hybrid <Landing Fuel, Total Travel>: This hybrid MILP formulation is an extended version of the ASP-TMA formulation for total travel time minimization (secondary indicator) in which Constraint (27) is added in order to force the landing aircraft trajectories with minimum fuel consumption (primary indicator). This hybridization may result in sub-optimal solutions for the total travel time minimization due to the larger travel time required by landing aircraft with the minimum fuel burned trajectories.

\[
\min \sum_{B=1}^{\text{total aircraft}} (t_{Bi} - t_{Bf})
\]

Combined <Delay, Total Travel>: This approach requires to solve two MILP formulations. First, we need to compute the value \( t_n^* \) of the optimal solution to the ASP-TMA formulation with maximum delay minimization (primary indicator). Second, we solve an extended ASP-TMA formulation with total travel time minimization (secondary indicator) in which we add the following constraint on the maximum delay: \( t_n \leq t_n^* \) (Constraint 30). In this way, we restrict the search to the only solutions that are optimal for the first formulation. The drawback is that we may remove the optimal solution of the original ASP-TMA formulation with total travel time minimization from the set of feasible solutions.

\[
\min \sum_{B=1}^{\text{total aircraft}} (t_{Bi} - t_{Bf})
\]

Combined <Total Travel, Delay>: This is another combined approach in which we first solve the ASP-TMA formulation with total travel time minimization (primary indicator). The value of the optimal solution to this formulation is named
\[ \sum_{B=1}^{\text{total aircraft}} (t_{ae(t)} - t_{ae}) \] . We then search for the optimal solution to the extended ASP-TMA formulation with maximum delay minimization (secondary indicator) in which we add the following constraint on the total travel time:
\[ \sum_{B=1}^{\text{total aircraft}} (t_{Bl} - t_{Bf}) \leq \{ \sum_{B=1}^{\text{total aircraft}} (t_{Bl} - t_{Bf}) \}^* \] (Constraint 31). As for the previous combined approach, the drawback is to restrict the search space of the ASP-TMA formulation with maximum delay minimization by possibly removing the optimal solution. This is necessary in order to preserve the solution optimality in terms of total travel time minimization.

\[ \min t_n \]

\[ \text{Constraints (17), (18), (19), (20), (21), (22), (23), (24), (25), (26)} \]

\[ \sum_{B=1}^{\text{total aircraft}} (t_{Bl} - t_{Bf}) \leq \{ \sum_{B=1}^{\text{total aircraft}} (t_{Bl} - t_{Bf}) \}^* \] (31)

Table 1 gives an overview of the proposed integrated approaches dealing with the optimization of multiple performance indicators. We report the primary (secondary) performance indicator with a ★ (◦).

<table>
<thead>
<tr>
<th>Type of Approach</th>
<th>Performance Indicators</th>
<th>Maximum Delay</th>
<th>Total Travel Time</th>
<th>Landing Travel Time</th>
<th>Landing Fuel Burned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lexicographic</td>
<td>&lt;Delay, Landing Travel&gt;</td>
<td>★</td>
<td>○</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lexicographic</td>
<td>&lt;Delay, Landing Fuel&gt;</td>
<td>★</td>
<td>○</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid</td>
<td>&lt;Landing Travel, Delay&gt;</td>
<td>○</td>
<td>★</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid</td>
<td>&lt;Landing Travel, Total Travel&gt;</td>
<td>○</td>
<td>★</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid</td>
<td>&lt;Landing Fuel, Delay&gt;</td>
<td>○</td>
<td></td>
<td></td>
<td>★</td>
</tr>
<tr>
<td>Hybrid</td>
<td>&lt;Landing Fuel, Total Travel&gt;</td>
<td>○</td>
<td></td>
<td></td>
<td>★</td>
</tr>
<tr>
<td>Combined</td>
<td>&lt;Delay, Total Travel&gt;</td>
<td>★</td>
<td>○</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Combined</td>
<td>&lt;Total Travel, Delay&gt;</td>
<td>○</td>
<td>★</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7 Computational experiments

This section presents the computational results on the integrated approaches presented in Sections 5 and 6. All experiments have been performed on a Quad-Core Intel Xeon E5 3.7GHz processor with 32 GB RAM, under OS X 10.10.3. The instances are based on real-world data of the Milano Malpensa (MXP) TMA.

Figure 2 shows a schematic view of MXP TMA. There are two interdependent runways (RWY 35L, RWY 35R), both used for landing and taking-off procedures. The MXP resources are the following: 3 airborne holding circles (resources 1-3, named TOR, MBR, SRN), 11 air segments for landing procedures (resources 4-14), 1 common glide path (resource 15), and 2 runways (resources 16-17). The common glide path resource includes two parallel air segments before the runways for which traffic regulations impose a minimum diagonal distance between landing aircraft in addition to a minimum longitudinal distance. In this work, we allow a maximum of three complete airborne holding circles for each landing aircraft.

Table 2 gives average information on the 35 instances used in the computational experiments. Column 1 presents the length of the time horizon of traffic optimization
(expressed in minutes), Columns 2-3 respectively the number of landing and taking-off aircraft, Columns 4-5 the maximum and average entrance delays (expressed in seconds). The entrance delay of an aircraft is defined as the difference between the actual and the expected entrance in the TMA. In this paper, the entrance delays are generated randomly with respect to a nominal (undisturbed) scenario (i.e. a reference timetable). Columns 6 and 7 report the total number of aircraft timing and sequencing decisions to be taken in each instance.

Table 2: Average data on the tested instances

<table>
<thead>
<tr>
<th>Time Horizon</th>
<th>Landing Aircraft</th>
<th>Take-off Aircraft</th>
<th>Max Entrance Delay</th>
<th>Avg Entrance Delay</th>
<th>Timing Decisions</th>
<th>Sequencing Decisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>14</td>
<td>6</td>
<td>1800</td>
<td>124</td>
<td>107</td>
<td>551</td>
</tr>
</tbody>
</table>

We next report the results obtained on the 4 integrated approaches based on a single performance indicator and on the 8 integrated approaches based on a multiple performance indicators. In total, we performed 420 experiments in order to evaluate all the proposed approaches on each problem instance.

### 7.1 Results on the single-indicator approaches

Table 3 presents the results obtained for the single-indicator approaches of Section 5. Each row reports average results on the 35 instances of Table 2. Column 1 indicates the evaluated approach, Columns 2-5 the maximum aircraft delay, the total aircraft travel time, the landing aircraft travel time and the landing aircraft fuel burned. In each row, the optimized indicator is highlighted in bold. All the solutions computed by each approach are optimal for the considered indicator (the one in bold). The computation time required by the approaches in order to proof the solution optimality is up to three minutes.

When comparing the various approaches in terms of the four performance indicators, we observe that no performance indicator is implicitly optimized by the single-indicator approaches. This leaves room for improvement by applying the multiple-indicator ap-
Table 3: Computational results when optimizing a single performance indicator

<table>
<thead>
<tr>
<th>Performance Indicator</th>
<th>Maximum Delay</th>
<th>Total Travel Time</th>
<th>Landing Travel Time</th>
<th>Landing Fuel Burned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Delay</td>
<td>59</td>
<td>12723</td>
<td>11050</td>
<td>1906</td>
</tr>
<tr>
<td>Total Travel Time</td>
<td>96</td>
<td>12539</td>
<td>10991</td>
<td>1958</td>
</tr>
<tr>
<td>Landing Travel Time</td>
<td>754</td>
<td>17269</td>
<td>10177</td>
<td>2433</td>
</tr>
<tr>
<td>Landing Fuel Burned</td>
<td>696</td>
<td>18783</td>
<td>13637</td>
<td>276</td>
</tr>
</tbody>
</table>

Table 4: Computational results when optimizing two performance indicators

<table>
<thead>
<tr>
<th>Type of Approach</th>
<th>Performance Indicators</th>
<th>Maximum Delay</th>
<th>Total Travel Time</th>
<th>Landing Travel Time</th>
<th>Landing Fuel Burned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lexicographic</td>
<td>&lt;Delay, Landing Travel&gt;</td>
<td>59</td>
<td>12723</td>
<td>10682</td>
<td>2174</td>
</tr>
<tr>
<td>Lexicographic</td>
<td>&lt;Delay, Landing Fuel&gt;</td>
<td>59</td>
<td>14107</td>
<td>11962</td>
<td>1288</td>
</tr>
<tr>
<td>Hybrid</td>
<td>&lt;Landing Travel, Delay&gt;</td>
<td>262</td>
<td>14197</td>
<td>10177</td>
<td>2433</td>
</tr>
<tr>
<td>Hybrid</td>
<td>&lt;Landing Travel, Total Travel&gt;</td>
<td>355</td>
<td>13878</td>
<td>10177</td>
<td>2433</td>
</tr>
<tr>
<td>Hybrid</td>
<td>&lt;Landing Fuel, Delay&gt;</td>
<td>501</td>
<td>17758</td>
<td>13637</td>
<td>276</td>
</tr>
<tr>
<td>Hybrid</td>
<td>&lt;Landing Fuel, Total Travel&gt;</td>
<td>585</td>
<td>17421</td>
<td>13637</td>
<td>276</td>
</tr>
<tr>
<td>Combined</td>
<td>&lt;Delay, Total Travel&gt;</td>
<td>59</td>
<td>12607</td>
<td>11029</td>
<td>1926</td>
</tr>
<tr>
<td>Combined</td>
<td>&lt;Total Travel, Delay&gt;</td>
<td>77</td>
<td>12539</td>
<td>10990</td>
<td>1951</td>
</tr>
</tbody>
</table>

From the average results of Table 4, none of the approaches outperforms the others in terms of all performance indicators. The value of the primary indicator (bold value) is always optimal for all the proposed approaches, while the quality of the secondary indicator (underlined value) depends on the specific approach. The unoptimized indicators

7.2 Results on the multiple-indicator approaches

Table 4 reports average results on the 35 instances of Table 2 solved by the multiple-indicator approaches of Section 6. Column 1 presents the type of integrated approach, Column 2 the primary and secondary indicators considered in the optimization process, Columns 3-6 the average results obtained in terms of the four performance indicators assessed in the current work. The bold [underline] values indicate the primary [secondary] performance indicator optimized by the corresponding approach (row of Table 4). Furthermore, all the bold values are proven optimal values for the corresponding indicator (column of Table 4). The average computation time required by the multiple-indicator approaches is also up to 3 minutes.
(neither bold or underlined values) are often far from their optimal value.

When assessing the different methods, the lexicographic approaches and the combined approaches compute, on average, a second-best solution in terms of their secondary indicator. This means that they are able to more adequately incorporate multiple performance indicators in the solution process compared to the hybrid approaches. However, the latter approaches focus on primary indicators related on landing aircraft, while the lexicographic and combined approaches deal with primary indicators related to all aircraft. For this reason, the hybrid approaches are less able to include secondary indicators dealing with all aircraft.

When looking at the specific performance indicators, the approaches primarily based on aircraft delay minimization perform well when combined with any kind of secondary indicator, both in terms of primary and secondary indicators. Differently, the approaches primarily based on landing fuel minimization poorly perform in terms of the aircraft delay or travel time minimization. Furthermore, the landing fuel minimization is not compatible with the landing travel minimization, since the former is to increase the travel time in the air segment of the TMA while the latter is to decrease the travel time in the air segment of the TMA. Finally, the approaches primarily based on landing travel minimization show a poor performance in terms of the secondary and unoptimized indicators, since they primarily focus on landing aircraft.

7.3 Overview of the results

Figure 3 presents a cumulative bar-plot of optimality gaps computed for the integrated approaches. The optimality gap related to each performance indicator is computed as $(UB - UB^*)/UB$ where $UB^*$ is the value computed by the examined approach and $UB^*$ is the proven optimal value. We recall that $UB^*$ is computed by the single-indicator integrated approaches within three minutes of computation.

![Figure 3: Cumulative optimality gaps for each integrated approach](image)

Regarding Figure 3, we have the following observations:
- Maximum delay and landing fuel burned are the indicators with the largest optimality gap when they are not primary indicators. These are often difficult to be minimized as secondary indicators.

- The approach with the lower largest optimality gap for all indicators is Lexicographic <Delay, Landing Fuel>. The aircraft delay minimization (primary indicator) favors the selection of quite good solutions in terms of total and landing travel times, while the worst indicator is landing fuel burned (secondary indicator). In general, the latter indicator is only improved when considered as a primary indicator.

- The approach with the lower cumulative optimality gap is Lexicographic <Delay, Landing Travel>. Three performance indicators are very good, while landing fuel burned is far from the optimal value. The low performance of the latter indicator is due the chosen secondary indicator (landing travel).

- The approaches with the higher cumulative optimality gap are the ones considering landing travel time as the primary indicator. This is mostly due to the fact that minimizing the landing travel time deteriorates the performance of maximum delay and landing fuel burned indicators.

- Among the single-indicator approaches, maximum delay minimization is the best approach, in terms of both the lower largest optimality gap for all indicators and the lower cumulative optimality gap.

8 Conclusions and future research

This paper deals with the real-time problem of efficiently scheduling take-off and landing operations at a busy TMA. This problem requires to consider complex features related to the accurate modeling of individual landing aircraft trajectories, and the simultaneous management of multiple aircraft in each TMA resource such that the safety rules between consecutive aircraft are respected while the aircraft timing and ordering decisions are optimized by minimizing aircraft delays, travel times and/or fuel consumption.

We consider the studied problem as a compound problem in which the ATOP computes the optimal landing trajectory for each aircraft and the ASP-TMA computes the optimal overall aircraft take-off and landing schedule in the TMA. These problems are formulated via state-of-the-art mathematical formulations and solved by dedicated software. Efficient approaches are then proposed for the integration of ATOP and ASP-TMA solutions, based on the optimization of single or multiple performance indicators.

Computational experiments have been performed on the MXP-TMA in case of disturbed traffic situations. The results show that single-indicator approaches are well-focused on a specific indicator but they are not able to implicitly optimize the other indicators. On the other hand, multiple-indicator approaches compute good trade-off solutions between several indicators. Concerning the various indicators, the minimization of the fuel burned clashes with the minimization of aircraft delay and travel time. This is due to the fact that the former indicator is to increase the travel time in the landing air segments, while the latter indicators are to decrease the travel time of both take-off and/or landing aircraft in the TMA.
Future research should be focused on the assessment of a larger set of traffic disturbances and traffic optimization scenarios in order to further evaluate the proposed integrated approaches. Other promising directions of research could be focused on the coordination of the TMA aircraft scheduling solutions with related air traffic management problems, such as the en-route, ground and gate scheduling problems.

References


APPENDIX

A numerical example is proposed in order to illustrate and further explain the alternative graph formulation of the investigated problem and some characteristics of the solutions computed by the following integrated approaches: maximum delay minimization, lexicographic <delay, landing fuel>, combined <delay, total travel>, and hybrid <landing fuel, delay>.

The proposed illustrative example considers four aircraft (J1, J2, J3, J4) travelling on the network of Figure 4. Specifically, J1, J2, J4 are landing aircraft, while J3 is a take-off aircraft. The network is composed of seventeen resources: 1–3 are the three airborne holding circles, 4–14 are the eleven landing air segments, 15 is the common glide path, 16–17 are the two runways. In particular, 16 is a mixed take-off and landing runway. The route assigned to each aircraft is the following: J1: 1-4-10-13-15-16-out; J2: 3-8-12-14-15-17-out; J3: 16-out; J4: 1-4-10-13-15-17-out. With out we mean the first resource outside the network.

Figure 4: The network and the route of each aircraft

Figure 5 shows the alternative graph G(N, F, A) of the example of Figure 4 for the lexicographic and combined approaches. A different color is used to highlight the sequence of nodes related to each aircraft (J1 is red, J2 is blue, J3 is orange, J4 is green). Each node (operation) in N (except for the start node 0 and the end node n) is identified by the following two-field code: (aircraft, resource). For example, node (J2,3) means aircraft J2 traverses resource 3. Each fixed directed arc in F is identified by a solid arrow, while each
alternative arc in $A$ is identified by a dotted arrow. The weight of each arc is depicted nearby the corresponding arrow.

Figure 5: Alternative graph $G(N, F, A)$ for the example of Figure 4

In $G(N, F, A)$ of Figure 5, the aircraft scheduling decisions are the following: airborne holding circle decisions for J1, J4 on resource 1 and J2 on resource 3 (the holding circle time can be either 0, 180 or 240); a sequencing order between J1 and J4 at the entrance in and at the exit from resources 4, 10, 13; a sequencing order between J1, J2 and J4 at the entrance in and at the exit from resource 15; a sequencing order between J1 and J3 (J2 and J4) on resource 16 (17). The minimum aircraft separation time is set to 42.

Figure 6 presents the graph $G(F, S)$ of an optimal solution for the example of Figure 4 in terms of maximum delay minimization. The start time of each operation is reported nearby the corresponding node and is depicted by using the same color of the node. In $G(F, S)$, exactly one arc is selected for each alternative pair. The following aircraft scheduling decisions have been taken: J1, J2 do not perform airborne holding circles while J4 performs holding circles for a total time of 180 in the airborne holding resource; J1 precedes J4 at the entrance in and at the exit from resources 4, 10, 13, and 15; the sequencing order at the entrance in and at the exit from resource 15 is J2, J1, and J4; J3 (J2) precedes J1 (J4) on resource 16 (17).

On the right bottom of Figure 6, we report the value of each performance indicator. The value of maximum delay minimization is 180. This is due to the additional time required by J4 in the airborne holding resource, in order to satisfy the minimum separation time constraints with J1 (J2) on the landing air segments (runway). The value of the other indicators is not optimized by the current approach.

Figure 7 presents the graph $G(F, S)$ of an optimal solution for the lexicographic $\langle$delay, landing fuel$\rangle$ approach. The graph presents the same aircraft sequencing (i.e. selection $S$) of the solution of Figure 6. However, the aircraft timing (i.e. the start time related to some nodes in $N$) is different. The latter difference is due to the landing fuel burned minimization process. When comparing the solutions of Figure 6 and 7 in terms of the four indicators (as shown in the right bottom tables), the maximum delay (primary indicator in both cases) is the same, while the landing fuel burned (secondary indicator of
the lexicographic <delay, landing fuel> approach) is strongly reduced in the solution of Figure 7. The remaining two indicators are considerably better in the solution of Figure 6, since the landing fuel burned minimization (by increasing the travel time) deteriorates the performance of the travel time indicators.

Figure 7: An optimal solution for the lexicographic <delay, landing fuel> approach

Figure 8 shows the graph $G(F \cup \{(n, 0)\}, S)$ of an optimal solution for the combined <delay, total travel> approach. $G(F, S)$ shows the following aircraft scheduling decisions: J4, J2 do not perform airborne holding circles while J1 performs holding circles for a total time of 180 in the airborne holding resource; J4 precedes J1 at the entrance in and at the exit from resources 4, 10, 13, and 15; the sequencing order at the entrance in and at the
exit from resource 15 is J2, J4, and J1; J3 (J2) precedes J1 (J4) on resource 16 (17).

Figure 8: An optimal solution for the combined <delay, total travel> approach

The right bottom table of Figure 8 reports the solution performance in terms of the four indicators. The value of maximum delay minimization (primary indicator) is 180. This is due to the additional time required by J1 in the airborne holding resource, in order to satisfy the minimum separation time constraints with J4 and J2 (J3) on the landing air segments (runway). The value of total travel time (secondary indicator) is 2789. The latter value is better than in the solutions of Figures 6 and 7. The primary and secondary indicators minimized by this approach have a positive effect on the landing travel time indicator, that is considerably better than in the previous solutions. On the other hand, as it is expected, the combined <delay, total travel> approach deteriorates the performance of landing fuel burned indicator.

Figure 9 shows the alternative graph $G'(N, F', A)$ for the example of Figure 4 when considering the trajectories minimizing the fuel burned. In this graph, the fixed arcs in $F'$ related to the minimum and maximum travel times allowed to traverse a landing air segment have the same weights in absolute value. These weights are set equal to the maximum travel time required by each aircraft in the landing air segment. The alternative graph $G'(N, F', A)$ is used in the hybrid <landing fuel, delay> approach. Figure 10 shows the connected graph $G(F', S)$ related to an optimal solution for the hybrid <landing fuel, delay> approach, that is characterized by the following aircraft scheduling decisions: J1 and J2 do not perform airborne holding circles, while J4 performs holding circles for a total time of 180 in the airborne holding resource; J1 precedes J4 at the entrance in and at the exit from resources 4, 10, 13, and 15; the sequencing order at the entrance in and at the exit from resource 15 is J2, J1, and J4; J3 (J2) precedes J1 (J4) on resource 16 (17). The sequencing decisions are the same as in the solution of Figure 6.

The right bottom of Figure 10 reports the value of the solution when evaluated with the four performance indicators considered. While the selection of the alternative arcs in
\( G(F', S) \) is the same as for the graph \( G(F, S) \) of Figure 6, the difference between the arc sets \( F \) and \( F' \) accounts for the difference between all the values of the four indicators. The performance indicator primarily optimized by the hybrid \(<\text{landing fuel, delay}>\) approach is landing fuel burned. This is the optimal value achievable for the proposed illustrative example. As for the secondary indicator of this hybrid approach, maximum delay is taken into consideration and the resulting value is 475. The latter value is clearly worst that the optimal value for \( G(F, S) \), since the set \( F' \) presents larger processing times than the set \( F \). In fact, J1 and J4 would suffer at least a delay equal to 295 in any scheduling solution related to \( G'(N, F', A) \). In the aircraft scheduling solution \( G(F', S) \), J4 is required to perform some holding circles in order to satisfy the minimum separation time constraints.

Figure 9: Alternative graph \( G'(N, F', A) \) for the example of Figure 4 with minimum fuel burned trajectories

<table>
<thead>
<tr>
<th>Performance Indicator</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Delay</td>
<td>475</td>
</tr>
<tr>
<td>Total Travel Time</td>
<td>4072</td>
</tr>
<tr>
<td>L. Travel Time</td>
<td>3181</td>
</tr>
<tr>
<td>L. Fuel Burned</td>
<td>65</td>
</tr>
</tbody>
</table>

Figure 10: An optimal solution for the combined \(<\text{landing fuel, delay}>\) approach
with J1 and J2, thus collecting an additional delay equal to the time spent in the airborne holding resource.