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State dependent spare part management policy for airport maintenance

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ABSTRACT

In this study we use a flexible policy of spare part management in the context of airports corrective maintenance activities and provide a new approach for mathematically evaluate the performances of systems with lateral transshipments in case of one-for-one ordering policies, transfer times and peaked demand. This study deals with the spare optimization of a large logistics company, in charge of the maintenance of 38 Italian Airports. Strict requirements of equipment operational availability and long repairing times make necessary to maintain high stock levels of spare parts, thus involving high inventory costs. While a two echelon inventory policy without lateral transshipments is currently applied, more flexible models with lateral transshipments might help to reduce inventory levels. We show the potential benefits deriving from implementing a single echelon policy with extensive use of lateral transshipments. The resulting model is a queueing network with zero buffer and lateral demand. The blocking probability of this network models the probability of spare part unavailability. We show that the probability of spare parts unavailability in the network can be easily computed using a Markov chain model. The system operational availability is then computed with two heuristic algorithms. The behavior of the two heuristics is very reliable if compared with that of the Markov chain model. Computational experiments, carried out with the proposed algorithms, show that the single-echelon policy under study allows to satisfy operational availability constraints with a significant reduction of spare stock levels with respect to the two echelon policy.

1. Introduction

Nowadays increase in air traffic leads to ever more complex systems for granting airport safety, as well as for supporting the correct execution of airport operations. Airports face every day the challenging task of maintaining high system availability of safety equipments, attained at a sustainable cost. As observed by several authors, see e.g. by [14], the logistics of spare parts differ from those of other materials in several ways. The equipments are characterized by remarkable costs, long repairing times and sporadic failures, which are difficult to forecast and cause relevant financial effects, due to the economical and legal implications of a lack of safety of airport operations.

When a failure takes place, failed components are removed and replaced with new spare parts. Safety reasons require prompt replenishments and are not compatible with long repairing times. In a common situation, the maintenance service is operated by specialized companies, and relevant elements such as operational availability, average or maximum replanishment time and penalties for late delivery are regulated by contract. The spare parts supply chain may typically involve at least three actors: airport authorities, logistics companies in charge of granting the quality of service, and equipment suppliers, which are responsible for supplying new components and/or repaired items.

In this study, we focus on the point of view of an intermediate logistics company facing the high cost of purchasing and managing spare parts at sustainable cost. We refer in particular to an Italian logistics company supporting the activity of 38 civil airports spread over the Italian territory. The company handles 17 warehouses for storing spare parts and manages the overall processes of purchasing, holding, ensuring that the overall reliability of safety equipments is always within contractual limits. The aim of the company is therefore to grant the prescribed quality of service at minimum cost.

We evaluate the benefits of a flexible policy of spare part management in this context and provide a new approach for modeling systems with lateral transshipments in case of one-for-one ordering policies, no negligible transshipment times and peaked demand. Specifically, in our system there are three possibilities for managing a failure. Each airport is associated to the nearest warehouse and, if spares are available in it, the failed component is replaced immediately with a spare and a request for a repair is then issued to the external supplier to restore the safety stock level. If no spares are available locally, a request for a spare is forwarded to the closest warehouse with available spares, so that the failed component can be replaced promptly. Also in this case a request for a repair is then issued to the external supplier to restore the safety stock level. Finally, if no spares are available in the 17 warehoses, a request for a repair or a new component is forwarded directly to the external supplier. In the latter case the failed component is replaced only after that the external supplier satisfies the request and this lead time is also regulated by contract. Unfortunately, lead times can range up to several months, which is not compatible with the high operational availability level of service which must be granted to the airport authorities.

The current logistics of spare parts at the considered company is based on a two-echelon system without lateral transshipment, depicted in figure 1 (a). Level of stock and geographical allocation of spare parts is obtained with the VARIMETRIC algorithm of [23]. However, lateral communication is a common practice in emergency situations, using couriers and overnight carriers to rapidly move parts to demand locations. Due to this reason, the

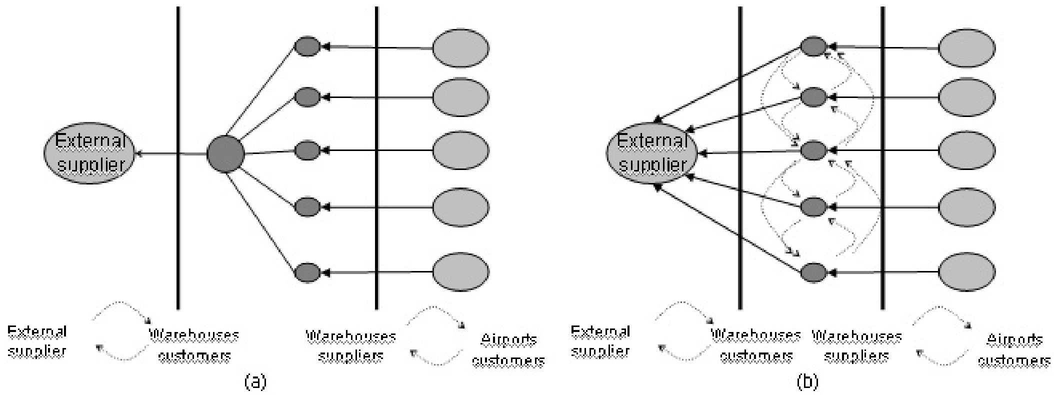

Figure 1: Logistics chain system: two echelon versus single echelon one

question arises whether more flexible policies, explicitly taking into account lateral transhipments, may yield better performance or not. The resulting single echelon system addressed in this paper is depicted in Figure 1(b). As observed, e.g., by [18], single echelon models with complete pooling are particularly effective for reducing both reaction time to stockouts and inventory levels.

The literature on spare part logistics is strictly related to the more general context of inventory management. A vast number of inventory models have been developed during the past decades. As observed in [28, 17] there are at least two main streams of reserch for approaching the modeling tasks. A first one follows multi-dimensional markovian approach [13] [26] and a second one is based on the METRIC approach of Sherbrooke [2] [3] [6] [1] [18] [28]. The first approach is based on the study of the system through flow balance equations and is computationally efficient when the number of warehouses and the level of stock is limited. The second approach is based on the Palm theory [23]. Among other studies concerning with transshipments policy we cite the work of [5].

We study a single echelon, multi location, single company inventory management policy based on item approach. Complete pooling of stock is permitted among the locations. The items of interest are expensive, repairable and subject to continuous review. Transshipment times are explicitly considered in computing the operational availability level, as well as the peaky nature of demand, due to lateral transshipments. When transfer times are negligible, such systems can be studied as aggregate Erlang loss systems [28]. However, in our case transfer times contribute to reduce the airport operational availability remarkably, and therefore cannot be omitted.

This paper is organized as follows. Section 2 formally defines the inventory models studied in this paper. Section 3 presents a simple heuristic procedure for allocating spare parts to the warehouses, used just as benchmark for successive comparisons. In Section 4 the reliability of the different models described in Section 2 is assessed based on a comparison with a Markov chain model. Then, the effectiveness of the single-echelon policy is compared with that of the currently used two-echelon one. Some conclusions follow in Section 5.

2. Model descriptions and assumptions

The model described is a single item, single echelon, N-locations, continuous review, one-for-one replenishment policy inventory system, which allows for lateral transshipments with complete pooling, state dependent transshipped demand and no negligible transfer times.

2.1 System processes description

When a failure occurs for some component at some airport, a demand for a new spare part is issued to the nearest warehouse. If spare parts are locally available, the component is immediately replaced in the airport using the stock on hand at the local warehouse. Then, a replenishment order to an external supplier, which can repair or give back a new item, is issued by the local warehouse to restore the local stock level for that specific component, after a replenishment time. When new spare parts are not available at the local warehouse, the demand is forwarded to the nearest location with available spare parts through a lateral transshipment. Then, the new destination warehouse will replace the component and issue a replenishment order to restore the stock level. If no warehouse in the network has available spares the demand must be satisfied by a direct shipment from the external supplier. In such a case we say that the warehouses network is blocked, and the failed component will be replaced only after that the supplier delivers the first repaired component.

The lead times for replenishment operations are random variables with known mean value, MTTR, exponentially distributed [3], which corresponds to assuming infinite capacity of repair shops and suppliers. The random failure process for each item in each airport is a random variable with exponential distribution and mean value equal to the MTBF (Mean Time Between Failures). The same kind of item in different operational conditions could have different MTBF due to damp, temperature, item age and other exogenous agents. It follows that the demand coming from the airports is poissonian. On the other hand, due to their dependence on the state of the network (i.e. stockouts and available spares), the lateral transshipments processes, and therefore the overall arrival process at each warehouse, follow a peaked distribution.

2.2 Basic model description and notation

We model the system dynamics with a queueing network, as follows. Each warehouse acts as a single queueing system without buffer, in which the number of servers equals the number of spares in the warehouse. The arrival process (i.e. demands for spare parts) is stochastic. Service times of each request equals the lead times needed to repair/replenish a spare part. The number of busy servers corresponds therefore to the number of outstanding orders of spare parts. We denote MCMT (mean corrective maintenance time) as the average amount of time an item is not available at an airport. With this model, the MCMT of each item corresponds to the time needed physically to substitute the spare as far as there are spares in the warehouse. If no spares are locally available, the request is forwarded to the closest warehouse with available spares and therefore a deterministic transfer time contributes to increase the MCMT. When there are no warehouses with available spares also the replenishment time from the external supplier has to be added to the MCMT: .

In order to formally define the problem we deal with in this paper, we need to introduce the following notation.

$I = 1, 2, ..., N$ index set of warehouses

λ_h $h = 1, ..., N$ arrival rate of failure processes of the item in operational systems connected to the warehouse h

$\lambda_h^{'}$ $h = 0, ..., N$ effective arrival rate taking into consideration transshipments

s_h $h = 1, ..., N$ number of spare parts of the considered item in warehouse h

P_{Btot} network blocking probability, demand rejected by the overall network

function expression for functional relationship

$P\{i\}$ probability of having available spares in a warehouse i

$P\{\neg i\}$ probability of not having available spares in a warehouse i

$\mu_j(n)$ state dependent service rate of warehouse j

T_{ji} transfer time for warehouse j to airport i (it includes T_s)

T_s substitution time, time needed to physically replace a failed item

$\pi(i, j)$ probability of the event: there are no spares in i and j is the closest warehouse with available spares

With this notation, the MCMT can be computed as follows.

$$MCMT = \sum_{i=1}^{N} \lambda_i \cdot T_s \cdot Pr\{i\} + \sum_{i=1}^{N} \lambda_i \cdot \sum_{j_i} \pi(i, j) \cdot T_{j_i} + (\sum_{i=1}^{N} \lambda_i) \cdot P_{Btot} \cdot (MTTR + OS)$$

T_s is negligible in our model. The performance measure established between the logistic company and the airport authorities is the operational availability, (OA), of the system, computed as follows [10] [23]:

$$OA = \frac{MTBF}{MTBF + MCMT}.$$

We study the following problem: find the minimum number of spares and their geographical allocation which achieves an airports operational availability greater then 99.6 %.

2.3 Mathematical model: Markov chain

Our model is equivalent to a queueing network with blocking, which can be studied using a Markov chain modeling approach [24].

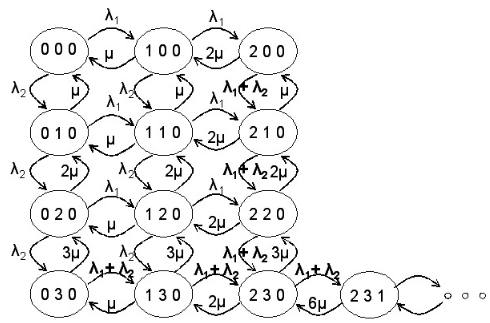

Figure 2: Example of Markov chain in a system of two warehouses

In the Markov chain a state is a multidimensional vector, in which the i-th element represent the number of requests at each warehouse as well as at the external supplier. Figure 2 shows the Markov chain for two warehouses, the first having two spares and the third having three available spares. With this model, in case of blocked network, the first repaired item returned by the the external supplier is used for replacing a failed item in some operative site, if any. There are direct transactions among states just in case of a single arrival event (i.e. a request for a spare) or a single departure event (i.e. the replenishment of a repaired item by the external supplier). $n + e_j$ is the state with an arrival in the j-th node in the network, due to a failure or a re-forwarded request. $n - e_j$ is the state with a departure from the j-th node in the network. The transition rates are the followings:

- q(n,$n + e_j$)=λ_j, if for each node i $n_i \neq s_i$
- q(n,$n + e_j$)=$\lambda_j + \sum_{x \in X, x \neq j} \lambda_x$, j has available spares and X is the blocked warehouses set, re-forwarding their requests to j, the closest warehouse with spares
- q(n,$n - e_j$)=$\mu_j(n)$

In principle one can derive the steady state probabilities for each state in the Markov chain by solving a linear system. However, the number of states may be exceedingly large when the number of warehouses and spares increases. Therefore we do not compute the steady state probability for each state, but limit ourselves to observe the P_{Btot}, which can be computed quite simply as follows. We can build an equivalent birth death system. We can group the states of the Markov chain, partitioning them. Nodes with multidimensional state vectors, whose elements sum to a same constant K, belong to the same group. In stationarity, the probability flows between sets constituting partitions of the state space are in balance (cross balance equations) [12]. Therefore it's possible to find a full set of partial balance equations for each group. We can apply partial balance properties, in particular the state aggregation property [22] [16], which substitutes each partition with an aggregated state and define a birth death process with transition rates equal to the average transition rate from states in one group to states in the successive one in the original process.

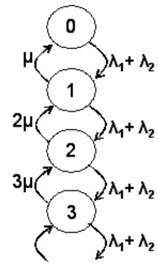

Figure 3: Example of equivalent birth death model

P_{Btot} is simply equal to the state probability of the equivalent node, whose state is equal to the sum of the allocated spares in each warehouse.

$$B = \sum_{i=1}^{N} s_h$$

$$\rho = \sum_{i=1}^{N} \lambda_i$$

$$Pr\{\sum_{i=1}^{N} n_i = K\} = \frac{\rho^K}{K!} \cdot e^{-\rho}$$

$$P_{Btot} = \sum_{i=B}^{\infty} Pr\{\sum_{j=1}^{N} n_j = i\}$$

$$P_{Btot} = 1 - \sum_{i=0}^{B-1} Pr\{i\}$$

2.4 Mathematical model: decomposition approach

The results obtained in the previous section does not allow to compute the OA of the system, since to this aim we also need the blocking probability of each warehouse. To compute these values we use two heuristic approaches, based on the study of each warehouse independently from the others. This approach can be used when the steady state can be expressed in product form. Product form queueing networks are based on the traffic equations; it is their structure that may introduce restrictive assumptions when capacity constraints are introduced. Traffic equations are important to define the mean effective arrival rate to each warehouse in the network. For the existence of a stationary distribution it is important that the routing is irriducible and that the effective mean rate entering each warehouse is not too high for the capacity of the warehouse. The arrival mean effective rates expresses the interactions among the warehouses: the evolution of each warehouse is therefore the same wether the arrivals come from airports, wether they come from other warehouses in the network. In networks with signals, arrivals triggering an immediate departure, the traffic equations are not linear. Such a queueing network has product form solution under

exponential assumptions and with a markovian routing [12]. Product form can be achieved also in case of state-dependent routing of customers between the service centers [7]. We don't get insight in this sense, rather we assume that state dependent traffic equations in our system have a solution and apply the decomposition approach.
The Markov chain model studied in this paper is unlikely to be in product form. However, as described in [7], product form networks provide the basis for many approximate algorithms to solve more general non product form ones. This approach is usually taken in similar kinds of networks in literature [20] [7] [9] [30]. Following up the decomposition approach the stockout events in different warehouses are assumed to be independent each other.

These considerations let us take into account the following mathematical model:

$$\begin{cases} \lambda' = \lambda + \sum_{j=1}^{N} \pi(i,j) \cdot \lambda'_j & \text{i=1,...,N} \\ P\{\neg i\} = function(\lambda'_i, s_i) & \text{i=1,...,N} \\ P_{Btot} = \prod_{i=1}^{N} P\{\neg i\} & \text{i=1,...,N} \end{cases}$$

In this model we have $N^2 + 2 \cdot N + 1$ variables. A_i terms let us normalizing state dependent reforwarding probabilities to one. In this model each warehouse is a G/M/s_i/0/∞ system. The functional relationship that relates the effective mean arrival rate to a warehouse and the number of allocated spares in it with its stockout probability is the same as a generalized Erlang blocking formula. In the following we'll take into consideration different arrival distributions, keeping the decomposition approach.

2.5 Decomposition approach: IPP and ERT

In the decomposition approach each warehouse is a G/M/s_i/0/∞ system and its stockout probability is the same as the generalized Erlang blocking formula. In what follows we'll take into consideration two ways to express the arrival process, capturing the demand variability due to stockouts, referred to as IPP (Interrupted Poisson Process), ERT (Equivalent Random Traffic) in [15]. However, while these models have been proposed and studied in the context of telecommunications networks for taking into account the peakedness of systems, in this paper we explore their applicability to the logistics context. To this aim, we make several adaptations to the models, described in the following, to better fit the specific application and compare their results with those of the Markov chain model, in order to evaluate the reliability of these models.

With the IPP model, the demand at each warehouse (including transshipments) can be adequately characterized by a simple renewal process. The inter arrival time distribution of an IPP is hyperexponential, commonly used in the literature to model high-variability arrival processes [25]. We have:

$$G(t) = a_1 \cdot (1 - e^{-\lambda_1 \cdot t}) + a_2 \cdot (1 - e^{-\lambda_2 \cdot t}) \tag{1}$$

where $a_1 + a_2 = 1$.

The a_i, λ_i, with $i = 1, 2$, are the parameters of the distribution.

In our case the probability density function is:

$$F(t) = a_1 \cdot \lambda_1 \cdot e^{-\lambda_1 \cdot t} + a_2 \cdot \lambda_2 \cdot e^{-\lambda_2 \cdot t} \tag{2}$$

The Laplace-Stieltjes transform is:

$$\phi(s) = \frac{a_1 \cdot \lambda_1}{s + \lambda_1} + \frac{a_2 \cdot \lambda_2}{s + \lambda_2} \tag{3}$$

The blocked fraction of the offered flow to each warehouse is computed using the generalized Erlang loss function.

$$B(S_i, a) = \frac{1}{\sum_{j=0}^{S_i} \frac{S_i!}{S_i - j! \cdot j!} \cdot \frac{1}{C_j(0)}} \tag{4}$$

where $C_0 = 1$ and $C_{-1}(\xi) = 1$ and:

$$C_j(\xi) = \prod_1^j \frac{\phi(i \cdot \mu + \xi)}{1 - \phi(i \cdot \mu + \xi)} \tag{5}$$

The total transshipped flow from a warehouse is viewed as the blocked fraction of the arrival process, in which requests come both from the airports and from the other warehouses. It is obtained by combining the characteristic moments of each arrival streams, due to failures or transshipments, under the assumption of independence. Carrying out the algebraic manipulations, we obtain the first three moments of the random variable describing the transshipped flow [20] [21].

$$\mu_1 = \frac{\alpha}{\mu} \cdot B(S_i, \frac{\lambda}{\mu}) \tag{6a}$$

$$\mu_2 = \mu_1 + \frac{\sum_{j=0}^{S_i} \frac{S_i!}{S_i - j! \cdot j!} \cdot \frac{1}{C_{j-1}(\mu)}}{\sum_{j=0}^{S_i} \frac{S_i!}{S_i - j! \cdot j!} \cdot \frac{1}{C_j(\mu)}} \cdot \mu_1 \tag{6b}$$

$$\mu_3 = 3 \cdot \mu_2 - 2 \cdot \mu_1 + 2 \cdot (\mu_2 - \mu_1) \cdot \frac{\sum_{j=0}^{S_i} \frac{S_i!}{S_i - j! \cdot j!} \cdot \frac{1}{C_{j-1}(2 \cdot \mu)}}{\sum_{j=0}^{S_i} \frac{S_i!}{S_i - j! \cdot j!} \cdot \frac{1}{C_j(2 \cdot \mu)}} \tag{6c}$$

The aggregated arrival stream, due to failures and transshipments, is again a renewal process and its characteristics are completely specified by the Laplace-Stieltjes transform of the inter arrival time distribution, G(t) [20]. A single stream is approximated through our renewal process, equating its three ordinary moments, μ_1, μ_2, μ_3 to the first three parametric moments characterizing G(t). We directly solve the non linear moment match equations, which leads us to achieve more reliable results with respect to the approximate computation used in [15]. To this aim we use a trust-region Newton method for uncostrained nonlinear equations and interior affine scaling approach for constrained optimization problems [8].

The steady-state distribution of each arrival process at each warehouse is obtained through an approximation scheme consisting of an iterative algorithm, described in Figure 4 At each iteration we estimate the parameter of the hyperexponential inter arrival time

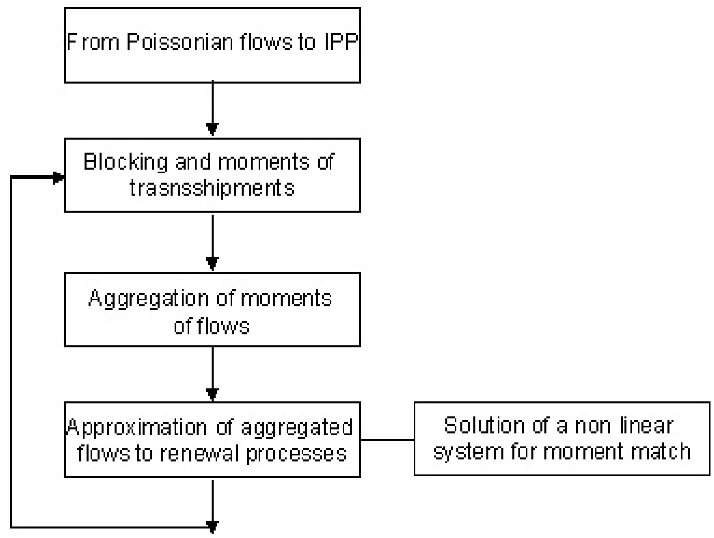

Figure 4: Iterative procedure for estimating steady state IPP parameters and steady state P_{Btot}

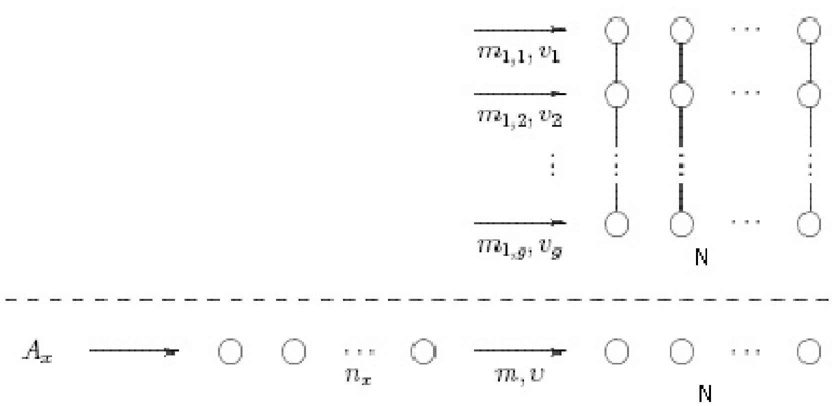

Figure 5: ERT: equivalent system

distribution through a moment match technique, taking into account the interactions among warehouses. We stop the iterations when the moments of the flows are within an ϵ.

With the ERT model, the demand at each warehouse (including transshipments) is characterized with a poisson process, whose mean and variance are computed as follows.

Specifically, the demand is viewed as the rejected flow coming from an equivalent queue with poissonian arrivals A_x and n_x servers. Figure 5 shows the equivalent system, totally composed by $n_x + N$ servers with unknown demand A_x. The equivalent system is a classical Kosten's system [15], in which Riordan's relationships among arrival process and mean value and variance of the transshipped flows are valid. We define:

m mean value of the transshipped flow

A_x mean value of the arrival process to the system

Z peakedness factor of the transshipped flow

v variance of the transshipped flow

$E_n(A_x)$ Erlang formula on the arrival process, whose mean value is A_x

$$m = A_x \cdot E_n(A_x) \tag{7a}$$

$$\frac{v}{m} = Z = 1 - m + \frac{A_x}{n_x + 1 - A_x + m} \tag{7b}$$

We determine the parameters of the equivalent system, the poissonian arrivals rate and the number of servers, by directly solving the moments equations . In this case the unknowns of the equivalent system are A_x and n_x. We solve this non linear system using a trust-region Newton method [8]. Finally we can apply the same formulas to characterize the mean value $m^{'}$ and peakedness factor $Z^{'}$ of the overflow of the total system, in which A_x is the arrival process and with $n_x + N$ servers.

$$m^{'} = A_x \cdot E_{n_x+N}(A_x) \tag{8a}$$

$$\frac{v^{'}}{m^{'}} = Z^{'} = 1 - m^{'} + \frac{A_x}{n_x + N + 1 - A_x + m^{'}} \tag{8b}$$

In the latter formulas the unknowns are now $m^{'}$ and $Z^{'}$.

3. Heuristic algorithm

Our heuristic procedure greedily allocates one spare part at a time and checks every time the operational availability of system until the contractual value (i.e. $99,6\%$) is reached. Its aim is that of avoiding allocation of too many parts to the same warehouse and distributing parts giving preference to warehouses with larger demand or allocating some spares to barycentric warehouses. This procedure, though simple, produces satisfactory results, checked by simulation [11]. In order to compute the OA associated to a given allocation of spare parts we need the blocking probability values of each warehouse. To compute these values, we follow three heuristic approaches:

- the IPP model of section 2.6;
- the ERT model of section 2.6;
- the greatest value for each warehouse obtained through IPP or ERT model;

Clearly, the third approach is more conservative than the others.

4. Computational experience

In this section we report on our computational experience. We carry out our experiments on the real data used by the logistic company to plan their stock levels, except for the transfer times for moving items between warehouses. Infact the VARIMETRIC algorithm only need to consider the transfer time between the central depot and each warehouse, while we allow transshipments among all pairs of warehouses. To this aim, we consider conservative values for the transfer times, which are equal to the actual travel time plus 24 hours of margin. These values are larger than those actually used by the VARIMETRIC algorithm. In this section, we report on our results with 30 items, corresponding to equipments for radars or weather sensors. The details on each item are described in Table 1. For each item the corresponding identification code and its approximate cost are reported in columns two and three. Columns four, five and six show the item repair rate and the item repair time (in months) and the MTBF in hours. In the last column the number of airports using the item and the total number of items in operation are given.
Table 2 evaluates the reliability of the IPP and ERT models, comparing the values of the P_{Btot} with the exact value computed with the Markov chain model. In Table 2 for each item P_{Btot} computed with IPP model and the corresponding OA are shown in column two and three. In the fourth and fifth column P_{Btot} computed with ERT model as well as the corresponding OA values are reported. In the last column we report the P_{Btot} computed with the Markov chain model.
The IPP model is quite reliable, especially when the total number of spares in the system is small. On the other hand the ERT appears more conservative with respect to the IPP model.

Table 3 report on a comparison between the allocation of spares obtained through our three heuristic approaches based on the single echelon policy and those allocated through VARIMETRIC algorithm. In the second and third column the total number of spares allocated by the VARIMETRIC algorithm and the corresponding network OA value computed with our policy for each item are reported. The total number of spares allocated with our heuristic, by using the IPP model for computing the OA values and the corresponding OA value are given in column four and five. Column six and seven show the total number of spares allocated with our heuristic, by using the ERT model and its OA value. The last two column show the same data for the third heuristic proposed approach.

These experiments show that the single echelon policy out-performs the two echelon one and the VARIMETRIC algorithmic approach, yielding an overall improvement of more than 29% of reduction in the total number of spare parts and spare parts purchasing cost savings of more than 19%. For the proposed testing set the stock value for VARIMETRIC allocations is almost 1.000.000 EUR, rather the same value for our heuristic allocations is almost 800.000 EUR. The data show that the operational availability reached is compatible with the contractual values, thus being always feasible.

5. Conclusions

In this paper we presented a new approach for analyzing inventory pooling of repairable spare parts in a air stocking point system. Computational experiments show that the single echelon policy in which the transshipment times are explicitly considered is effective in allocation of spare parts for repairable, expensive items. The proposed OA computation, based on conservative values of transfer times, is computationally efficient and effective with respect to the VARIMETRIC algorithm, thus demonstrating the high potential of single echelon policy with respect to the two echelon policy in the spare parts management for airport maintenance operations. A greedy heuristic procedure allocates a minimal number of spare parts in the system reaching operational availability values, which are compatible with the contractual values, thus being always feasible. Further research should investigate closed form expressions for computing the blocking probability of single warehouses, thus avoiding any approximation.

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Item	Code/Name	Cost	Repair time	MTBF	number airports	number items
1	Magnetron	$\simeq$ 26000 €	3	16000	4	8
2	Was-425-AH	$\simeq$ 4000 €	3	61000	4	9
3	UT021	$\simeq$ 1800 €	3	45000	14	20
4	Was-425-AH-C	$\simeq$ 4000 €	3	52000	1	2
5	U1501	$\simeq$ 6000 €	3	12000	11	19
6	1390	$\simeq$ 10000 €	3	132000	4	8
7	138109A	$\simeq$ 2000 €	3	191000	5	16
8	AAC0004/01	$\simeq$ 2000 €	3	138000	1	4
9	ADH-3COM	$\simeq$ 2000 €	3	15300	1	2
10	138053A	$\simeq$ 1000 €	3	118000	2	6
11	138054A	$\simeq$ 700 €	3	226000	11	42
12	138055A	$\simeq$ 900 €	3	126000	13	28
13	138056A	$\simeq$ 2000 €	3	79000	8	10
14	138084A	$\simeq$ 6000 €	3	262000	15	41
15	DTS12G	$\simeq$ 1000 €	3	22000	1	1
16	GILL1390	$\simeq$ 4000 €	3	74000	1	4
17	HMP35D	$\simeq$ 3000 €	3	159000	15	25
18	PA-9870	$\simeq$ 1000 €	3	27000	1	1
19	PMT16A	$\simeq$ 4000 €	3	13000	1	2
20	PTB100A	$\simeq$ 2000 €	3	101000	8	18
21	PTB220	$\simeq$ 4000 €	3	81000	3	5
22	QMW101	$\simeq$ 900 €	3	13000	1	1
23	US621	$\simeq$ 2000 €	3	17000	7	11
24	UM-5505	$\simeq$ 2000 €	3	607000	14	22
25	UM-9730B	$\simeq$ 2000 €	3	26000	2	3
26	US120	$\simeq$ 1000 €	3	38000	6	10
27	US121	$\simeq$ 3000 €	3	200000	7	11
28	US620	$\simeq$ 1000 €	3	94000	6	10
29	USD21	$\simeq$ 600 €	3	172000	14	26
30	USN21	$\simeq$ 2000 €	3	76000	11	25

Table 1: Characteristic data for 30 items

Item	IPP P_{Btot}	IPP AO	ERT P_{Btot}	ERT AO	Markov P_{Btot}
1	$1.9 \cdot 10^{-10}$	0.998	$9.9 \cdot 10^{-9}$	0.997	$1.3 \cdot 10^{-8}$
2	$9.5 \cdot 10^{-5}$	0.999	$3.2 \cdot 10^{-4}$	0.993	$6.0 \cdot 10^{-6}$
3	$1.5 \cdot 10^{-17}$	0.995	$4.2 \cdot 10^{-15}$	0.991	$6.9 \cdot 10^{-20}$
4	0.00232	0.999	0.01574	0.998	0.00233
5	$8.0 \cdot 10^{-24}$	0.978	$4.1 \cdot 10^{-21}$	0.945	$2.1 \cdot 10^{-35}$
6	$1.3 \cdot 10^{-6}$	0.999	$1.8 \cdot 10^{-5}$	0.999	$4.9 \cdot 10^{-7}$
7	$9.7 \cdot 10^{-9}$	0.999	$5.2 \cdot 10^{-7}$	0.999	$2.7 \cdot 10^{-8}$
8	0.0187	0.987	0.0957	0.938	$7.8 \cdot 10^{-5}$
9	0.00367	0.999	0.0238	0.997	$1.1 \cdot 10^{-4}$
10	0.0021	0.999	0.0081	0.997	$1.9 \cdot 10^{-4}$
11	$2.4 \cdot 10^{-19}$	0.999	$6.2 \cdot 10^{-16}$	0.998	$1.4 \cdot 10^{-20}$
12	$1.0 \cdot 10^{-21}$	0.999	$1.1 \cdot 10^{-17}$	0.998	$2.2 \cdot 10^{-19}$
13	$8.2 \cdot 10^{-13}$	0.999	$5.9 \cdot 10^{-17}$	0.998	$4.6 \cdot 10^{-13}$
14	0.00433	0.999	0.00433	0.999	0.00436
15	0.0059	0.998	0.0059	0.998	0.0060
16	$2.6 \cdot 10^{-26}$	0.999	$2.5 \cdot 10^{-19}$	0.999	$2.0 \cdot 10^{-23}$
17	0.00293	0.999	0.00293	0.999	0.00295
18	0.0114	0.995	0.0114	0.995	$6.4 \cdot 10^{-4}$
19	$1.4 \cdot 10^{-15}$	0.999	$7.7 \cdot 10^{-14}$	0.999	$7.1 \cdot 10^{-15}$
20	$5.7 \cdot 10^{-5}$	0.999	$7.6 \cdot 10^{-4}$	0.999	$1.1 \cdot 10^{-5}$
21	0.0114	0.995	0.0258	0.990	$6.4 \cdot 10^{-4}$
22	$8.0 \cdot 10^{-24}$	0.978	$4.1 \cdot 10^{-21}$	0.945	$2.1 \cdot 10^{-35}$
23	$6.1 \cdot 10^{-13}$	0.999	$5.3 \cdot 10^{-10}$	0.999	$2.8 \cdot 10^{-12}$
24	0.0172	0.998	0.0668	0.990	$1.4 \cdot 10^{-6}$
25	$8.1 \cdot 10^{-4}$	0.998	0.00284	0.994	$6.2 \cdot 10^{-6}$
26	$1.0 \cdot 10^{-8}$	0.997	$1.8 \cdot 10^{-6}$	0.994	$9.6 \cdot 10^{-9}$
27	$2.2 \cdot 10^{-13}$	0.999	$1.3 \cdot 10^{-11}$	0.999	$8.7 \cdot 10^{-13}$
28	$3.1 \cdot 10^{-45}$	0.999	$4.8 \cdot 10^{-29}$	0.997	$6.4 \cdot 10^{-19}$
29	$5.6 \cdot 10^{-28}$	0.999	$1.5 \cdot 10^{-26}$	0.999	$1.6 \cdot 10^{-24}$
30	$3.5 \cdot 10^{-24}$	0.987	$9.0 \cdot 10^{-16}$	0.976	$2.6 \cdot 10^{-22}$

Table 2: comparison among models: P_{Btot} and AO

Item	VARIMETRIC		IPP		ERT		Third approach	
	spares	OA	spares	OA	spares	OA	spares	OA
1	10	0.998	7	0.996	10	0.996	10	0.996
2	5	0.999	2	0.997	4	0.998	4	0.998
3	20	0.995	17	0.996	17	0.996	17	0.996
4	2	0.999	2	0.998	2	0.998	2	0.998
5	46	0.978	37	0.996	38	0.996	38	0.996
6	5	0.999	2	0.999	3	0.998	3	0.999
7	6	0.999	2	0.998	3	0.998	3	0.998
8	4	0.987	3	0.999	4	0.999	4	0.999
9	3	0.999	2	0.999	2	0.997	2	0.997
10	3	0.999	2	0.999	2	0.997	2	0.999
11	16	0.999	3	0.996	7	0.997	7	0.997
12	16	0.999	4	0.996	10	0.997	10	0.997
13	10	0.999	2	0.996	5	0.996	5	0.996
14	6	0.999	2	0.998	3	0.998	3	0.998
15	5	0.999	2	0.999	3	0.998	3	0.999
16	2	0.999	2	0.999	2	0.999	2	0.999
17	2	0.998	2	0.998	2	0.998	2	0.998
18	17	0.999	2	0.999	4	0.996	4	0.996
19	2	0.999	2	0.997	2	0.997	2	0.997
20	3	0.995	3	0.999	3	0.998	3	0.998
21	12	0.999	3	0.997	7	0.997	7	0.998
22	4	0.999	2	0.999	2	0.997	2	0.999
23	3	0.995	3	0.999	3	0.999	3	0.999
24	4	0.998	1	0.998	2	0.998	2	0.998
25	5	0.998	3	0.998	4	0.999	4	0.999
26	9	0.997	9	0.996	9	0.996	9	0.996
27	8	0.999	2	0.997	2	0.997	2	0.998
28	8	0.999	2	0.996	3	0.996	3	0.998
29	17	0.999	3	0.996	14	0.996	14	0.996
30	25	0.987	21	0.996	24	0.996	24	0.996

Table 3: comparison among allocation heuristics