Optimal allocation plan for
distribution centers of a frozen
food company

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ABSTRACT

In this paper we analyze the distribution system of an Italian company operating in the ice cream and frozen food market. In particular, we address the problem of optimally allocating products demand to distribution centers spread over the Italian territory and develop a mixed integer programming model. We present our computational experience in which the optimal solution is compared with the actual distribution policies and show how to use our model as a decision support tool for the company management.
1 Introduction

Logistics activities include transportation, inventory maintenance, order processing, warehousing, and materials handling. These activities provide a bridge between producers of goods and the market consumers who are separated by time and distance. It has been estimated that logistics activities represent a large portion of a firm’s costs. (According to the National Council of Physical Distribution, already in 1978, people and goods transportation expenses in the US were about the 15% of the US GDP). In fact, recently, interest has increasingly grown in the field of optimal material flow to improve market competition through the reduction of production cost and the satisfaction of customers, and much research has dealt with the optimization of the design and management of distribution systems (see, for instance, \([1, 4, 5, 6, 9]\))

This work describes the results of a study that was undertaken to analyze and improve the distribution system performance of an Italian company, Sagit S.p.A., operating in the ice cream and frozen food market. In particular, the problem of optimally allocating products demand to distribution centers spread over the Italian territory is addressed. The company is the leader in the frozen food and ice cream Italian market with a market share of \(35 - 57\%\) for ice cream and \(41\%\) for frozen food (Source: Nielsen, 1999). In 1998, turnover was around \(1'400\) millions of Euro; sales were around \(420'000\) pallets of product, corresponding to moving about \(1'500'000\) cube meter of product on \(500\) trucks (TIR) per day.

Frozen food and ice cream market in Italy is characterized by very few competitors. This is mostly due to the extremely high stock buildings and transportation costs. It has been shown that the quality and duration of frozen food products depends on two main factors. The perishability of a product depends on the temperature food is stored at; in fact, the lower is the stocking temperature the longer the product lasts. Moreover, food organoleptic characteristics are better preserved if the temperature is kept in a narrow range during the stocking period.

Hence, stock buildings are actually “huge freezers” that must guarantee a steady and extremely low temperature and an aseptic environment. The loading and unloading operations from/to warehouses must be performed in a cold and protected environment. Therefore, transportation and handling costs are usually higher than in other types of market. In this context, logistics decisions play a fundamental role and have relevant impact on operations management expenses.

The object of our analysis may be viewed as a divergent serial multi-echelon distribution system with non-identical warehouses. Non-identical warehouses are typical of practical situations, the differences among warehouses being, for instance, the shipment lead times, and the capacities. The initial research on multi-echelon inventory models is generally attributed to Clark and Scarf ([3]), who studied a \(n\)-echelon serial system operating under periodic review ordering policies. The problem most commonly addressed in the literature is the depot-warehouse problem (two-echelon system). A typical assumption is that the warehouses are all identical, recently though, in [1] allocation policies when the warehouses are not identical are considered.

Here we consider a distribution system consisting of three stages, where different depots are to supply several non-identical warehouses (called Primary Distribution Centers, PDC) which in turn supply other warehouses (called Secondary Distribution Centers, SDC). Each SDC then serves a number of retailers, thus defining a retail distribution
system with a single warehouse. This general problem of allocating inventories from a central warehouse to different retailers has been widely studied in the literature. See, for instance [8], where a complete and extensive survey of references to this problem is provided. See also [7, 2].

In this paper, we model the problem as a deterministic capacitated multi-period multi-commodity location-allocation Mixed Integer Program. We do not address the problem that deals mainly with order policies, in fact, we consider lead times as negligible and address commodity flow optimization. We assume that periodic demands are known and the production volume is sufficient to supply them.

The paper is organized as follows. The next section is introductory and gives a detailed description of the distribution system of the company. In Section 3 a Mixed Integer Programming model for the system is introduced. Section 4 presents the results of our computational experience and finally, in Section 5, some conclusions are drawn.

2 Distribution network

In this section we describe the actual distribution system and policies of the company. In Figure 1 we sketch the distribution network and flows. The network consists of four levels: Supply sites, Primary Distribution Centers, Secondary Distribution Centers, and Retailers. Four types of different distribution flows take place between levels.

![Distribution network diagram]

Figure 1: Distribution flows.

2.1 Product types

The different types of products can be clustered into four main categories, depending on the destination market (grocery stores or cafeterias/restaurants) and on the format characteristics of product pack.

Product quantities are expressed in a volume unit denoted as IP (Industrial Pallet), which is equal to 100 × 120 × 180 cm³. The actual quantity of product contained in a IP may substantially vary depending on the final destination market. For instance, as regards ice creams, in grocery stores they are sold in eye-catching and robust packs, called multipacks, containing from 4 to 12 servings. Those for cafeterias, where ice creams are stored in display cabinets and served one at a time, are larger and may hold roughly 20 to 40 servings.

Thus, we distinguish the following four commodities: (i) ice cream for grocery stores, (ii) ice cream for cafeterias or restaurants, (iii) frozen food for grocery stores, and (iv) frozen dessert for cafeterias and restaurants.
2.2 Logistic poles and distribution flows

The finished products are made in 35 production plants located in Italy and abroad (by the so called co-packers). In our model, these facilities have been aggregated into two production poles located in Naples (which includes the plant in Cagliari also) and Latina, plus an extra eleven “external supply sites” which are the collection points of goods coming from abroad. In Figure 2 and Table 1 production data of 1998 are reported.

![Graphs showing production trends for different months and locations.]

Figure 2: Production trends in 1998 for frozen food (a) produced at Latina (dark grey) and by copackers (light grey), and for ice cream (b) produced at Napoli, Cagliari (dark grey) and by co-packers (light grey).

<table>
<thead>
<tr>
<th>Supply Sites</th>
<th>Ice Cream for Grocery Stores</th>
<th>Ice Cream for Cafeterias</th>
<th>Frozen Food for Grocery Stores</th>
<th>Frozen Food for Cafeterias</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Latina</td>
<td>0</td>
<td>0</td>
<td>119186</td>
<td>5884</td>
</tr>
<tr>
<td>2. Napoli</td>
<td>74988</td>
<td>128083</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3. Verona</td>
<td>580</td>
<td>991</td>
<td>18135</td>
<td>895</td>
</tr>
<tr>
<td>4. Ancona</td>
<td>0</td>
<td>0</td>
<td>4372</td>
<td>216</td>
</tr>
<tr>
<td>5. Teramo</td>
<td>0</td>
<td>0</td>
<td>374</td>
<td>18</td>
</tr>
<tr>
<td>6. Modena</td>
<td>0</td>
<td>0</td>
<td>3859</td>
<td>191</td>
</tr>
<tr>
<td>7. Ascoli</td>
<td>0</td>
<td>0</td>
<td>693</td>
<td>34</td>
</tr>
<tr>
<td>8. Ortucchio</td>
<td>0</td>
<td>0</td>
<td>8454</td>
<td>417</td>
</tr>
<tr>
<td>9. Matera</td>
<td>0</td>
<td>0</td>
<td>4608</td>
<td>227</td>
</tr>
<tr>
<td>10. Milano</td>
<td>0</td>
<td>0</td>
<td>669</td>
<td>33</td>
</tr>
<tr>
<td>11. Courmayeur</td>
<td>1596</td>
<td>2726</td>
<td>25694</td>
<td>1268</td>
</tr>
<tr>
<td>12. Genova</td>
<td>464</td>
<td>793</td>
<td>416</td>
<td>21</td>
</tr>
<tr>
<td>13. Livorno</td>
<td>0</td>
<td>0</td>
<td>2890</td>
<td>143</td>
</tr>
</tbody>
</table>

Table 1: Annual production data at supply sites per each commodity.

From the supply sites the finished products are distributed to 14 Primary Distribution Centers where quality checks are performed and stocks are formed. Goods must be transshipped to PDC’s as soon as they are available at the supply sites (in fact, one may assume that supply sites do not have stocking facilities). We refer to the material flow from supply sites to PDC’s as the supply flow.

The PDC’s are very large warehouses where products are stored at $-27^\circ C$ in an aseptic environment. The 14 primary distribution centers can be aggregated into 4 clusters where
two elements in the same cluster are so close that transportation costs between them become negligible, as it appears in practice. In our model we consider these clusters as the actual PDC’s. For each PDC the maximum stock capacity, the handling and stocking costs are known and fixed throughout the whole time horizon (see Table 3). Notice that, as products are perishable, their picking follows a First-In-First-Out policy, thus not allowing the utilization of the whole available space. As a consequence, there is a gap around 20% between nominal (available volume) and actual capacities of Primary Distribution Centers.

Next, products are transferred to about 200 Secondary Distribution Centers (SDC), which are located all over the Italian national territory. This distribution flow is referred to as the primary flow. Like the PDC’s, we aggregated the SDC’s reducing their number to 90. Aggregated demand data for all the 90 SDC’s are known for each time period and for each commodity.

<table>
<thead>
<tr>
<th>Region</th>
<th># of SDC’s</th>
<th>Total Demand</th>
<th>Max SDC Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Val D’Aosta</td>
<td>1</td>
<td>2194</td>
<td>2194</td>
</tr>
<tr>
<td>Piemonte</td>
<td>6</td>
<td>20287</td>
<td>6142</td>
</tr>
<tr>
<td>Lombardia</td>
<td>11</td>
<td>87116</td>
<td>19941</td>
</tr>
<tr>
<td>Triveneto</td>
<td>12</td>
<td>34038</td>
<td>9834</td>
</tr>
<tr>
<td>Liguria</td>
<td>3</td>
<td>9457</td>
<td>5315</td>
</tr>
<tr>
<td>Emilia Romagna</td>
<td>8</td>
<td>27256</td>
<td>9672</td>
</tr>
<tr>
<td>Toscana</td>
<td>7</td>
<td>21600</td>
<td>10015</td>
</tr>
<tr>
<td>Marche</td>
<td>2</td>
<td>10745</td>
<td>8065</td>
</tr>
<tr>
<td>Umbria</td>
<td>2</td>
<td>6584</td>
<td>4396</td>
</tr>
<tr>
<td>Lazio</td>
<td>6</td>
<td>55018</td>
<td>26401</td>
</tr>
<tr>
<td>Abruzzo</td>
<td>4</td>
<td>10569</td>
<td>4991</td>
</tr>
<tr>
<td>Campania</td>
<td>8</td>
<td>36277</td>
<td>13386</td>
</tr>
<tr>
<td>Basilicata</td>
<td>2</td>
<td>5331</td>
<td>3974</td>
</tr>
<tr>
<td>Puglia</td>
<td>6</td>
<td>21311</td>
<td>10709</td>
</tr>
<tr>
<td>Calabria</td>
<td>2</td>
<td>13388</td>
<td>12530</td>
</tr>
<tr>
<td>Sicilia</td>
<td>9</td>
<td>34781</td>
<td>12675</td>
</tr>
<tr>
<td>Sardegna</td>
<td>1</td>
<td>12877</td>
<td>12877</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>98</strong></td>
<td><strong>40919</strong></td>
<td><strong>26401</strong></td>
</tr>
</tbody>
</table>

Table 2: Aggregated annual demand data.

In Table 2, we summarize the total annual demand of Secondary Distribution Centers for each region of the Italian national territory. Note that, in our model there may be more than one SDC in a region. In the last 5 years, market annual growth has been steady and it has been measured around 1% per year. Such a stable behavior allows the use of deterministic models based on historical aggregated data, and as a consequence, production planning can be effectively optimized by company management.

In Figure 3 demand trends during the year for ice cream and frozen food are shown. Note that remarkable demand changes may occur from period to period. In particular, ice cream demand has a peak during warmer months whereas frozen food shows an almost opposite trend: for instance, people buy frozen vegetables when fresh product is not available. On the other hand, on the whole, production has an almost steady flow all over the year (see Figure 2). In this case, it is clear how PDC’s act as buffers between demand and production flows.
The Company management, mainly for administrative purposes, imposes that each SDC demand must be satisfied by only one PDC. In order to adjust stock levels, however, products stored at a PDC may also be transshipped to another PDC, which has to be temporarily replenished. This particular kind of distribution flow between PDC’s is referred to as *inter-pole flow* (see Figure 1). Finally, products are sent to the actual retail sites (grocery stores/cafeterias). This distribution flow is called *secondary flow*. It is assumed that the opportunity for transshipment between SDC’s and stock returns to the warehouse are precluded.

Optimization of secondary flow is not analyzed in this paper. As we have already observed, each SDC defines a retail distribution system with a single warehouse, which can be optimized efficiently with standard techniques (see [8]).

### 2.3 Costs

In this work we deal with the optimization of the supply and primary flows that minimize a total distribution cost function. The latter objective function takes into account:

1. stocking costs summed over all the PDC’s, all the time periods and all the commodities (costs for different commodities may differ at the same PDC because of the package characteristics);

2. handling costs at all PDC’s and during each time period, and

3. transportation costs for the three types of distribution flows (supply, primary, interpole: for the cost per kilometer to transship a pallet, from a supply point to a PDC, may differ from the cost for moving the same pallet from a PDC to a SDC).

In Table 3 handling and stocking costs for all PDC’s are reported.

Primary flow involves handling of about 800'000 IP per year, which takes place on approximately 700 trucks. Almost all these vehicles may host around 65 IP in a temperature controlled (refrigerated) environment. The company entrusts transportation services to third parties called *vectors* which are small private business owning the vehicles. For almost all vectors Sagit S.p.A. is in fact their unique client. As a consequence, Sagit may exert a strong influence on vectors in terms of service quality checks (including compliance
<table>
<thead>
<tr>
<th>PDC</th>
<th>Stocking Cost (Euro × month/IP)</th>
<th>Handling Cost (Euro/IP)</th>
<th>Actual Capacity (IP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Alano</td>
<td>7.4</td>
<td>10.0</td>
<td>38000</td>
</tr>
<tr>
<td>2. Naples</td>
<td>7.5</td>
<td>10.3</td>
<td>30500</td>
</tr>
<tr>
<td>3. Milan</td>
<td>11.2</td>
<td>14.6</td>
<td>4700</td>
</tr>
<tr>
<td>4. Latina</td>
<td>10.6</td>
<td>14.6</td>
<td>46350</td>
</tr>
</tbody>
</table>

Table 3: Actual capacities, storage and handling costs at aggregated PDC’s.

with pick-up and delivery dates) and transportation fees. At the moment, Italian standards on freight transportation establish upper and lower bounds on cost per kilometer. This cost depends on vehicles capacity (though not by the truck load) and decreases as the total distance traveled grows. The actual fees paid by the company are the object of a negotiation between the company and the vectors: typical, smaller fees are obtained assigning larger amounts of service to the same vector/truck. It is important to observe that unit transportation fees may substantially differ from each other, depending on the particular region of the country. During the negotiation phase, also truck routes are decided, i.e., the distribution centers visiting sequence and the associated pick-up and delivery dates. On the other hand, the driver may always decide autonomously the actual vehicle journey. As a matter of fact, at the moment, a prescriptive decision model, including the optimization of this particular function, is unusable. Leaving appropriate decisional autonomy to the vectors and maintaining good terms with them has a greater strategic and social priority than optimizing vehicles routings. Moreover, analysis on the field highlighted that vectors decisional autonomy often produces “locally” good solutions.

In conclusion, our model uses historical aggregated data for the transportation costs of one pallet per kilometer. All these data have been obtained obtained a posteriori by measuring, for each type of transportation flow, the average bargained costs and the average number of pallets traveling per vehicle.

3 Optimization Model

We now describe the mathematical model developed in our study. The problem has been formulated as a mixed integer program. Plans are made on a monthly base with a time horizon of one year. We thus consider 12 time periods \( t = 1, \ldots, 12 \) and maintain the four commodities \( i = 1, 2, 3, 4 \) introduced above. Input data are in order.

Model Data

Production data (quantities of product supplied) at each supply site \( p \) and demand data at each SDC \( j \) are known and deterministic for all the commodities \( i \) and for each period \( t \). In our experiments, we use production data summarized in Table 1 which refer to 1998 production. In Sagit S.p.A., production planning is decided at a higher hierarchical level and thus is independent of distribution policies. Actually, as we have already observed, due to market stability, demand trends may be predicted with a very good approximation and production data may be considered as fixed.

For each PDC \( j \), \( j = 1, 2, 3, 4 \), the maximum stock capacity \( Q_j \), the handling cost \( H_j \), and stocking cost \( S_j \) are given. Note that these data do not vary from period to period.
As already mentioned, we aggregated the 200 secondary distribution centers into 90 SDC’s. Aggregated demand data for all the 90 SDC’s are given for each time period. In particular, for each SDC, \( h = 1, \ldots, 90 \), monthly demand for the \( i \)-th commodity \( D_{ht} \) is given, for all \( i = 1, 2, 3, 4 \), and \( t = 1, \ldots, 12 \).

Unit transportation costs are indicated by \( T_{uv}^1 \), \( T_{uv}^2 \), and \( T_{uv}^3 \), where the pair \((u, v)\) represents the origin-destination pair sourcing point-PDC, PDC-SDC, and PDC-PDC, respectively.

Variables

We use the following decision variables:

- \( x_{jh} \) is a binary variable indicating whether SDC \( j \) is assigned to PDC \( h \) or not. We get 360 such variables which are denoted as Primary Flow variables. Note that these variables completely model the primary distribution flow since each SDC demand is entirely satisfied by a single PDC.

- \( z_{jt}^i \) is a nonnegative real variable measuring the quantity (in terms of number of pallets) of the \( i \)-th commodity stored at the \( j \)-th PDC during month \( t \). These are denoted as Stock variables. We have 192 such variables.

- \( u_{pjt}^i \) is a nonnegative real variable indicating the quantity of the \( i \)-th commodity sent from sourcing point \( p \) to the \( j \)-th PDC during month \( t \). There are 2496 variables \( u_{pjt}^i \) (denoted as Supply Flow variables).

- \( y_{jkt}^i \) is a nonnegative real variable indicating the quantity of the \( i \)-th commodity sent from the \( j \)-th PDC to the \( k \)-th PDC during month \( t \). There are 768 variables \( y_{jkt}^i \) (Inter-Pole Flow variables).

In conclusion we have 3816 variables, about 10% of which are integer variables.

Constraints

Constraints concern the following items. There are 192 Stock Continuity constraints (one for each stock variable) which relate each stock variable with demands from SDC’s and supplies from other PDC’s and supply sites:

\[
\begin{align*}
  z_{jt}^i &= z_{jt-1}^i + \sum_{p=1}^{13} w_{pjt}^i - \sum_{h=1}^{90} D_{ht}^i x_{jh} - \sum_{k=1}^{4} y_{jkt}^i + \sum_{k=1}^{4} y_{kjt}^i,
\end{align*}
\]

for all \( j = 1, \ldots, 4 \), \( t = 1, \ldots, 12 \), \( i = 1, \ldots, 4 \).

Moreover, we have 48 Stock Capacity constraints at PDC’s:

\[
\sum_{i=1}^{4} z_{jt}^i \leq Q_j \quad \text{for all } j = 1, \ldots, 4, \ t = 1, \ldots, 12.
\]

Then, there are 624 Supply Flow constraints which impose that goods “produced” at supply sites must be immediately shipped to the PDC’s:

\[
\sum_{p=1}^{13} w_{pjt}^i = P_{pt}^i \quad \text{for all } j = 1, \ldots, 4, \ t = 1, \ldots, 12, \ p = 1, \ldots, 13.
\]
There are 90 Assignment constraints which indicate that each SDC is supplied with every commodity from a single PDC:

$$\sum_{j=1}^{4} x_{jh} = 1 \quad \text{for all } h = 1, 2, \ldots, 90.$$  

**Objective Function**

Our objective is to minimize the total management cost of the logistics chain. This cost is composed by:

1. **stocking costs** which will be expressed as

$$\sum_{i=1}^{4} \sum_{j=1}^{4} \sum_{t=1}^{12} S_{j,t}^i z_{j,t}^i;$$

2. **handling costs**, which can be expressed as

$$\frac{1}{2} \sum_{j=1}^{4} \sum_{i=1}^{4} \sum_{t=1}^{12} H_j \left( \sum_{p=1}^{13} \sum_{t=1}^{12} \left( u_{pjt} + 90 D_{hj}^i x_{j,t}^i \right) \right);$$

3. and transportation costs for the three origin-destination pairs, which can be expressed as:

$$\sum_{i=1}^{4} \sum_{j=1}^{4} \sum_{t=1}^{12} \left( \sum_{p=1}^{13} u_{pjt}^i T_{pjt}^i + \sum_{h=1}^{90} D_{hj}^i x_{j,t}^i T_{hj}^i + \sum_{k=1}^{4} \left( k \neq j \right) T_{jk}^i + \sum_{k=1}^{4} \left( k \neq j \right) \left( T_{jk}^i + \frac{1}{2} H_j \right) \right).$$

**4 Computational Results**

In this section we describe our computational experiences which consisted of two main experiments: the first is the optimization of the actual scenario; the second group of experiments concerned sensitivity analysis useful to point out future directions for the management of the company. In particular, the following three scenarios have been considered: (i) increased volumes of production at supply sites and demand at SDC’s (and varied stock capacity at PDC’s); (ii) exclusion of Milan PDC; (iii) decreased transportation costs. In this sense our model appeared to be an effective decision support system.

Experiments highlighted a good formulation for the problem. In fact, in the first experiments the solution for the linear relaxation provides an integer optimal solution. For the other experiments, the gap between the linear relaxation and integer program optimal values is always below 3%.

All the data used in our experiments refer to the monthly 1998 production. We used Microsoft Excel 7.0 on a Pentium 100, 32 Mb RAM, under Windows NT, as I/O data interface, and Cplex 3.0 on an IBM RISC 6000 under AIX 3.0 o.s., as MIP solver.
<table>
<thead>
<tr>
<th>Costs</th>
<th>Fraction of the total cost</th>
<th>Savings (w.r.t. partial costs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocking</td>
<td>27%</td>
<td>3.1%</td>
</tr>
<tr>
<td>Handling</td>
<td>16%</td>
<td>1%</td>
</tr>
<tr>
<td>Transportation</td>
<td>51%</td>
<td>4.9%</td>
</tr>
<tr>
<td>Inter-PDC transp.</td>
<td>6%</td>
<td>40%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100%</strong></td>
<td><strong>5.7%</strong></td>
</tr>
</tbody>
</table>

Table 4: Comparison between distribution policies.

### 4.1 Optimization of the actual scenario

In this experiment we used real data to test the model described in Section 3 and compared the solution found with the actual policies of the company.

Table 4 illustrates costs savings obtained by using our model compared to the actual distribution policy. The total cost saving is equal to more than 1’500’000 Euro, which corresponds to 5.7% of the total distribution cost. The total cost saving of the system is composed by 245’000 Euro for stocking costs (corresponding to a 3.1% decrease with respect to actual stocking expenses), 49’000 Euro for handling costs (that is a 1% decrease with respect to actual handling expenses), and 750’000 Euro for transportation costs (about 4.9% decrease with respect to actual transportation expenses). Moreover, a consistent decrease in inter-pole transportation costs can be observed (around 40%, corresponding to 596’500 Euro).

![Demand Volumes](chart1.png) ![Number of SDC's](chart2.png)

Figure 4: Comparison between the actual and optimized scenarios: number of SDC’s and demand volumes allocated at PDC’s.

Few comments on the differences between actual scenario and the solution suggested by our model follow. Figures 4 and 5 illustrate the comparison between the two distribution policies in terms of demand quantities and number of SDC’s allocated.

The allocation of SDC’s to PDC’s is very similar in the two cases: the assignment is the same for 85% of the SDC’s. However, not surprisingly, due to its lower handling and storage costs, our MIP model makes a greater use of Naples PDC, and a smaller use of Latina PDC. On the other hand, demand volumes allocated at PDC’s differ a lot in the
actual scenario and in the model. For instance, note how, in the model, the load of Latina PDC decreases dramatically.

In Figure 6 the stock levels at PDC’s for each month are compared. In the actual scenario the stock levels follow demand trends in a similar fashion (they all decrease during summer and increase during winter). On the other hand, in the optimized scenario, clearly PDC’s capacities are exploited depending on their costs. Therefore, Alanno and Naples stock levels are kept at their maximum, while Latina and Milan reflect demand trends.

4.2 Other experiments

For what concerns the sensitivity analysis experiments, our model has been extensively used to give suggestions that can help the Company management in congested demand scenarios. The experiments are described hereafter.

Increase of production rate

Our model has been tested in the modified scenario in which production rate and demand has been increased of 20%, keeping the other data fixed.

We may observe that:

- total costs increases by 14.4%: this is mainly due to an increased rate of transshipment between PDC’s (59%);
- minor variations in the allocation SDC-to-PDC (less than 2.2%) which shows an inherent robustness of the solution found by our model.

It is clear how in this case, it may be profitable to increase stock capacity at some PDC. In order to determine which PDC’s capacity is the best to expand, we may proceed as follows.

Remove all PDC’s capacity constraints and solve the resulting MIP model. It is reasonable that the PDC with higher stock volumes is the one whose expansion would mostly benefit the system. In our experiments, it turned out that 50% of the demand was allocated to Alanno PDC (30% to Naples and less than 20% to Milan and Latina).

On the basis of the preceding results an additional experiment has been conducted. At Alanno PDC capacity has been expanded of 15% and handling and stocking costs increased by 10%. (The latter data has been provided by the company management as a consequence of PDC expanded capacity). In this case we obtain a substantial cost saving (around 6.5%, i.e., 2'137'000 Euro) with respect to the case with augmented production-demand and PDC’s capacities unmodified.

Exclusion of Milan PDC

Another experiment regarded the exclusion of the primary distribution center of Milan. The reason of this analysis is that Milan PDC has the lowest capacity and the highest stocking and handling costs. Moreover, it can be observed that during demand peaks, because of its small capacity, it is forced to take advantage of Alanno PDC warehouse, thus augmenting inter-places flow. In the model, excluding Milan PDC can be done by setting all the corresponding variables to 0.
Figure 5: Graphical comparison between the actual and optimized scenarios in terms of SDC-PDC allocation.
Figure 6: Comparison of PDC’s stock levels between the actual and optimized scenarios.

The experiments highlighted the following system behavior: although stocking costs on the whole decreased (3.7%), total cost increased of 5.2% (around 1’495’000 Euro), due to a dramatic increase of handling costs (17.6%, corresponding to 846’000 Euro) and to a 6% increase in transportation costs. Figure 7 illustrates the experimental results in terms of demand quantities and number of allocated SDC’s.

Figure 7: Allocation of SDC’s and demand volumes to PDC’s with the exclusion of Milan.

**Reduction of transportation cost**

As radical changes in the Italian standards on transportation are foreseen in the next future, transportation office at Sagit S.p.A. estimates a reduction of about 10% in the transportation costs for the company. On this basis, the last set of experiments concerned the analysis of a modified scenario where the cost for transshipment (for all the types of origin-destination pair) are reduced at the same rate.

By running our model, the observed decrease of the objective function is around 5.4% (that is more than 1’540’000 Euro) of the total cost with respect to the actual scenario.

As one may expect, largest savings (16.5%) concern transportation expenses, thus reducing their incidence on the total cost from 50.6% to 44.6%.
Stocking costs remain almost the same but, together with an increase of the inter-pole flow (and of the cost associated with it) which is around 25%, handling costs rise up to more 10% than in the actual scenario. This is not surprising, as it is now more convenient to transship goods from one PDC to a less expensive one. Correspondingly, the growth in handling costs partially counteract this trend.

The differences in terms of demand quantities and number of allocated SDC’s between the scenario with reduced transportation costs and the actual one can be seen in Figure 8.

![Graphs showing Demand Volumes and Number of SDC's](image)

Figure 8: Allocation of SDC's and demand volumes to PDC's with reduced transportation costs.

## 5 Summary and Conclusions

This paper has presented an approach, based on mixed integer programming, to the optimization of a distribution system for an Italian company operating in the ice cream and frozen food market. The problem consisted on allocating the demand of specific logistic poles spread all over the Italian territory (denoted as secondary distribution centers) to some other (primary) distribution centers which are, in turn, supplied by some other production poles.

Data were provided by the company management and refer to the production and demand detailed per months of one year (1998). A set of experiments has been designed in order to compare the performance of the actual and the optimized distribution system but also to obtain significant information about possible action coping with predictable changes in the demand scenario. In this sense our model appeared to be an effective decision support system as it may provide useful indications to the company management.

Other investigations into different distribution policies are underway. Discussion with the company indicated that a decision support system that allows the production planning module to interact with an optimization model for distribution logistics would in fact be extremely useful. Future research will therefore concern the design of algorithms for production planning using (iteratively) our model as a routine for the demand allocation.
References


