Natural properties of abductive hypotheses in three-valued logic*

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Summary

This paper shows some interesting properties of Kleene's three-valued logic in relation to abductive reasoning. A semantical characterization of abductive explanations is proposed, based on the notion of minimal three-valued model. This establishes a relation between the minimization problem in abductive reasoning and three-valued semantics, in the same sense as non-monotonic reasoning deals with minimization in two-valued semantics.

1 Introduction

Abductive reasoning is that form of unsound reasoning that infers a conclusion ϕ from premises $\phi \supset \psi$ and ψ. Its importance in Artificial Intelligence lies both on the fact that it reflects some forms of commonsense reasoning [15, 16, 19], where causes for events are to be hypothesized, and on its link with diagnostic reasoning [14]. In fact, in the AI literature, abductive reasoning is a way to solve problems where an observed event ϵ is not explained

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by the presently adopted theory Σ and an explanation for ϵ has to be looked for. So, assuming that $\Sigma \not\models \epsilon$, if γ is an explanation for ϵ in the context of Σ, then ϵ is a logical consequence of $\Sigma \cup \{\gamma\}$.

The first explicit studies on abductive reasoning are due to Pierce, although some authors attribute the origins of the notion to Aristotele. The development of modern logic highlighted the fundamental duality between abduction and deduction, due to the fact that, if Σ is a logical theory and ϵ an observed fact, then for any γ (in any logic where sentences can "cross" the logical entailment symbol $\models$):

$$\Sigma, \gamma \models \epsilon \quad iff \quad \Sigma, \neg\epsilon \models \neg\gamma$$

Thus, as pointed out since the earliest papers on modern abduction [13, 2], any deductive system that can be used to generate consequences can also be used to perform abduction.

A peculiar feature of abductive reasoning is that, usually, explanations are required to respect some fundamental conditions, in order to be accepted as "interesting". The three main restrictions imposed on explanations γ for an observation ϵ in the context of a theory Σ are the following ones:

(i) γ is consistent with Σ, i.e. $\Sigma \not\models \neg\gamma$.

(ii) γ is a minimal explanation, i.e. for all γ' that are explanations of ϵ in the context of Σ ($\Sigma, \gamma' \models \epsilon$), if $\gamma \models \gamma'$ then $\models \gamma' \equiv \gamma$.

(iii) γ has some restricted syntactical form; for example, it is a prenex formula whose matrix is a conjunction of literals.

Condition (i) rules out trivial explanations, condition (ii) redundant ones and condition (iii) is motivated by the fact that if γ is an explanation for ϵ in the context of Σ, that is consistent with Σ and minimal, then γ is equivalent to $\Sigma \supset \epsilon$. In this work any γ satisfying condition (iii) and such that $\Sigma, \gamma \models \epsilon$ is considered as an explanation for ϵ in the context of Σ. When an explanation is required to be either minimal or consistent with Σ, it will be explicitly stated.

Most proof theoretic methods for performing abduction are based on resolution (see for example [11, 12, 2, 8, 20]). An exhaustive approach was introduced by [12] and recently described again in [11], that consists in generating all the resolvents of the clause form of the theory and the negation of the observation, selecting the minimal ones and negating them.

In order to reduce the set of hypotheses that are generated, [2] constructs hypotheses from the dead ends of linear resolution proofs. The minimal ones are filtered after the generation of all the solutions. This method is not complete: it does not compute all the minimal explanations for the abductive problem, but only the explanations that the authors call *basic*, i.e. explanations that cannot be further explained in any non-trivial way in the context of the same theory. For example, if $\Sigma = \{q \supset p, r \supset q\}$, then there are three minimal and consistent conjunctions of literals that explain the observation p, namely p itself, q and r. However, p is considered trivial (and it is also non basic); q is not basic, because r is a non-trivial explanation for it in the context of Σ. And in fact, $\neg q$ is not a dead-end of a resolution from the clause form of Σ and $\neg p$.

This corresponds to the usual reading of clauses in a logic program as rules, where the head and tail of a rule have a different epistemological meaning: the tail is an explanation of the head. In this sense, if Σ is written as a logic program, containing the clauses $p \leftarrow q$ and $q \leftarrow r$, then clearly q is an intermediate explanation for p, because it can be further explained by r. The basicality condition, however, seems to be meaningful only when it is stated in relation with theories represented by logic programs. In fact, if the theory Σ considered before is rewritten as $\{\neg q \vee p, \neg r \vee q\}$, there seems to be no reasons for preferring the explanation r to q. Indeed, in some cases q may even be preferred to r: for example, because the set of consequences of $\Sigma \cup \{q\}$ is smaller, i.e. q is a weaker assumption that, when added to Σ, derives p. The abduction method based on performing resolution steps as far as possible, however, does not calculate q as one of the minimal explanations for p in Σ.

Finally, it is to be noted that there are abductive problems that do not admit non-trivial and consistent explanations (a simple example is when $\Sigma = \{p\}$ and $\epsilon = q$) and problems that have only non-basic minimal explanations. Consider for example the theory $\Sigma = \{q \supset p, q \equiv r\}$ and the observation p. There are exactly two minimal (and non-trivial) explanations for this problem that are consistent with Σ, i.e. q and r, but neither of them is basic. In this case, resolution *tout court* from Σ and $\neg p$ does not terminate.

Other methods based on linear resolution are defined, for example, in [20] and [8]. In [20] an extension of SLD-resolution is used, coping also with negation as failure. [8] defines a modification of linear resolution that is complete w.r.t. the generation of *interesting* consequences. Most of the works on abduction in the context of logic programming, however, are strongly influenced by the linear resolution view, even in the definition of the basic concepts involved.

The above sketchy observations suggest that investigating the nature of abduction outside the context of resolution based logic programming still deserves attention, in order to characterize abductive explanations without tailoring them to any fixed method of computation.

In this work, we propose a semantical characterization of abductive explanations based on Kleene's three-valued logic. Minimal three-valued models are in fact shown to have interesting properties in relation to abductive reasoning. This highlights a new connection between abductive and nonmonotonic reasoning: some formalisms for nonmonotonic reasoning are based on the selection of the best - according to some conditions - among minimal models of the theory (generally two valued models are considered, but see also [5]). In this work we show that abductive explanations are to be chosen among the minimal three-valued models of the abductive problem (the definition of general criteria for selecting the best explanation is still an open question). This establishes a relation between the minimization problem in abductive reasoning and three-valued semantics, in the same sense as non-monotonic reasoning deals with minimization in two-valued semantics.

The fundamental results are stated for propositional logic first, and then extended to a first order restricted language.

2 Kleene's three-valued logic

We consider a first order logical language $\mathcal{L}^-$, without function symbols with *arity* > 0. Atomic formulae, literals, ground expressions and sentences are defined as usual. A *clause* is a purely universal prenex sentence, whose matrix is a disjunction of literals: $\forall y_1 \cdots \forall y_r (L_1 \vee \cdots \vee L_n)$. A clause will be identified with the set of its literals and set operations on clauses are defined as usual. Analogously, a conjunction of literals ψ is said to be included in another conjunction of literals ψ' if all the literals in ψ occur also in ψ'. Two disjunctions (conjunctions) are considered equal - and the equality sign '=' will be used - if they disjoin (conjoin) sets of equal elements. A *fundamental clause* is a non tautological clause where no literal is repeated. A *fundamental conjunction* is a non contradictory conjunction of literals where no literal is repeated.

A ground sentence ϕ of $\mathcal{L}^-$ is said to be in *conjunctive normal form (CNF)* (resp. *disjunctive normal form (DNF)*) if either ϕ is a fundamental disjunction (resp. conjunction), or ϕ is a conjunction (resp. disjunction) of two or more fundamental disjunctions (resp. conjunctions), none of which is a logical consequence of another one. Consequently, the transformation of ϕ into CNF (DNF) requires the elimination of tautologies (contradictions) and elements that include other elements. We convene that the empty disjunction is equivalent to the atom $false$ and the empty conjunction to the atom *true*.

In the following we assume that a theory Σ is a finite set of purely universal sentences of $\mathcal{L}^-$. Σ is clearly equivalent to a set of clauses in $\mathcal{L}^-$.

A *Herbrand interpretation* for the language $\mathcal{L}^-$ is an interpretation of $\mathcal{L}^-$ whose domain is the Herbrand universe of $\mathcal{L}^-$. The *ground instantiation* Σ^* of a theory Σ is the set of all ground instances of formulae in Σ, obtained by substituting elements of the Herbrand universe for variables in all possible ways. Clearly, Σ^* is finite and a Herbrand interpretation of Σ is a model of Σ iff it is a model of Σ^*. Hence, over its Herbrand universe, each purely universal theory is equivalent to a propositional theory.

A *three-valued Herbrand interpretation* M for the language $\mathcal{L}^-$ is a set of ground literals, where no atom A occurs both positive and negative. The interpretation of an atom A in M, $\overline{M}(A)$, is a mapping onto the set of truth values $\{T, *, F\}$, where T and F correspond to classical truth and falsity, and $*$ is meant to capture the notion of absence of information. $\overline{M}(A)$ is defined by stipulating that:

$$\overline{M}(A) = \begin{cases} T & \text{if } A \in M \\ * & \text{if } A \notin M \text{ and } \neg A \notin M \\ F & \text{otherwise, i.e. if } \neg A \in M \end{cases}$$

In Kleene's three-valued logic, it is convened that $F < * < T$. The interpretation $\overline{M}$ is extended recursively to a truth valuation on the set of all ground sentences of the language by establishing that [9]:

$$\begin{array}{rcl} \overline{M}(\neg\phi) & = & \begin{cases} T & \text{if } \overline{M}(\phi) = F \\ * & \text{if } \overline{M}(\phi) = * \\ F & \text{otherwise, i.e. if } \overline{M}(\phi) = T \end{cases} \\ \overline{M}(\phi \wedge \psi) & = & min(\overline{M}(\phi), \overline{M}(\psi)) \\ \overline{M}(\phi \vee \psi) & = & max(\overline{M}(\phi), \overline{M}(\psi)) \\ \overline{M}(\phi \supset \psi) & = & max(\overline{M}(\neg\phi), \overline{M}(\psi)) \end{array}$$

A three-valued Herbrand interpretation M is a model of a purely universal set of sentences Σ ($M \models_3 \Sigma$) if $\overline{M}(\phi) = T$ for each $\phi \in \Sigma^*$. When $\Sigma = \{\phi\}$, we shall write $M \models_3 \phi$ instead of $M \models_3 \{\phi\}$. A formula ϕ is a three-valued consequence of Σ ($\Sigma \models_3 \phi$) iff for any three-valued Herbrand interpretation M, if $M \models_3 \Sigma$ then $M \models_3 \phi$.

If I and J are three-valued Herbrand interpretations, we say that J informationally extends I iff $I \subset J$. A three-valued model I of Σ is *minimal* if for no model J of Σ, $J \subset I$.

Kleene's logic has the following properties:

a. Kleene's three-valued logic has no logical theorems: for any ϕ, $\not\models_3 \phi$. In fact, no formula is true in the empty model.[1]

b. Contradictions are never true, i.e. $A \wedge \neg A$ has no three-valued models.

c. If $I \subseteq J$ and ϕ is a sentence, then $I \models_3 \phi$ implies $J \models_3 \phi$. (Persistence [10]).

As two-valued models are also three-valued models, $\Sigma \models_2 \phi$ implies $\Sigma \models_3 \phi$, but the converse is not always true. In particular, $CNF(\phi) \models_3 \phi$

[1] In order to have both tautologies and a deduction theorem the logic is often extended with a weak negation (see [4, 5]). Another way for closing Kleene's three-valued logic under classical consequence, is by means of the supervaluation truth definition, originally introduced by Bas van Fraassen [7]

may be false. For example, if $\phi = p \supset (p \wedge q)$, so that $CNF(\phi) = \neg p \vee q$, and $M = \{q\}$, then $M \models_3 CNF(\phi)$ but $\overline{M}(\phi) = *$. However, $\phi \models_3 CNF(\phi)$, $DNF(\phi) \models_3 \phi$ and $\phi \models_3 DNF(\phi)$.

If M is any finite Herbrand interpretation, then $Th(M)$ denotes the conjunction of the literals in M. Note that any Herbrand interpretation of a purely universal formula in $\mathcal{L}^-$ is finite. The following properties specify the relation between $\models_3$ and the classical consequence relation. If M is any finite Herbrand interpretation and ϕ, ψ are formulae, then:

1. If $M \models_3 \phi$ then $Th(M) \models \phi$.
2. If ϕ is not valid and $Th(M) \models \phi$, then $M \models_3 CNF(\phi)$.
3. If ψ is not valid and $\phi \models \psi$, then $\phi \models_3 CNF(\psi)$.
4. If $\phi \models_3 \psi$, then $\phi \models \psi$

The above properties show that Kleene's semantics is sensitive to normal form transformations and tautology elimination, that are the basic operations for checking propositional derivability. It is this feature that makes it valuable for the computation of abductive hypotheses.

3 Abduction in the ground case: prime implicants, prime implicates and minimal explanations

Let ϕ be a sentence in $\mathcal{L}^-$. A *prime implicant* of ϕ is a conjunction of literals ψ such that $\psi \models \phi$ and if ψ' is any conjunction of literals such that $\psi' \models \phi$ and $\psi \models \psi'$, then $\psi' \models \psi$. The notion of propositional prime implicant is due to Quine [17, 18] and it has been extended to first order only recently [11]. It is of central interest in abduction. In fact the set of the prime implicants of $\Sigma \supset \epsilon$ coincides with the set of the minimal explanations for ϵ in the context of Σ.

In the propositional case the computation of the set of all prime implicants of a sentence is a matter of syntactical transformations of such a sentence into normal forms, as the following theorem states.

Theorem 1 *Let ψ be any ground formula and $\psi_1, \ldots, \psi_n$ all the prime implicants of ψ. Then $\psi_1 \vee \ldots \vee \psi_n = DNF(CNF(\psi))$.*

(Note that the above symbol $=$ is equality, not logical equivalence.) This is not surprising: deduction itself, in the propositional case, is very much a matter of pure syntactical transformations of formulae (checking whether ψ is valid amounts to checking whether $CNF(\psi) = true$, or, in terms of refutation, whether $DNF(\neg\phi) = false$. See also [1]).

The following theorems state the fundamental connection between three-valued semantics and abduction.

Theorem 2 *Let ϕ be a non valid ground sentence of $\mathcal{L}^-$, $M_1, \ldots M_n$ all the minimal three-valued models of $CNF(\phi)$, and $\psi_1, \ldots, \psi_n$ all the prime implicants of ϕ. Then $\psi_1 \vee \ldots \vee \psi_n = Th(M_1) \vee \ldots \vee Th(M_n)$.*

A method to construct the three-valued minimal models of a set of purely universal clauses, based on [6], can be given and it will be described in a longer paper.

A *prime implicate* of a formula ψ is a clause ϕ such that $\psi \models \phi$ and if ϕ' is a clause such that $\psi \models \phi'$ and $\phi' \models \phi$ then $\phi \models \phi'$. An obvious relation between prime implicants and prime implicates is that ψ is a prime implicant of ϕ iff $\neg\psi$ is a prime implicate of $\neg\phi$. So, Theorem 2 already gives a method for determining the set of prime implicates of a propositional formula ϕ, i.e. it is $\{\neg Th(M) | M$ is a minimal three-valued model of $\neg\phi\}$. The following result establishes a direct relation between prime implicates and minimal three valued models.

Theorem 3 *Let ϕ be a non valid ground sentence of $\mathcal{L}^-$, $M_1, \ldots M_k$ all the minimal three-valued models of ϕ, and $\xi_1, \ldots, \xi_n$ all the prime implicates of ϕ. Then $\xi_1 \wedge \ldots \wedge \xi_n = CNF(Th(M_1) \vee \ldots \vee Th(M_k))$.*

Theorems 2 and 3 give two dual methods for computing the set of minimal explanations for a ground event ϵ in the context of a ground theory Σ:

Corollary 1 *Let $\Sigma = \{\sigma_1, \ldots, \sigma_k\}$ be a set of ground sentences and ϵ a ground observation, such that $\Sigma \not\models \epsilon$. The set of minimal explanations for ϵ in the context of Σ is $\{Th(M) | M$ is a minimal three-valued model of $CNF((\sigma_1 \wedge \ldots \wedge \sigma_k) \supset \epsilon)\}$.*

Corollary 2 *Let Σ be a set of ground sentences and ϵ a ground observation, such that $\not\models \Sigma \cup \{\neg\epsilon\}$ (i.e. it is not the case that Σ is valid and ϵ is contradictory - a very trivial case is ruled out). Let $M_1, \ldots, M_r$ be the minimal three-valued models of $\Sigma \cup \{\neg\epsilon\}$ and let $CNF(Th(M_1) \vee \ldots \vee Th(M_r)) = \xi_1 \wedge \ldots \wedge \xi_m$. The set of all the minimal explanations for ϵ in the context of Σ is $\{\neg\xi_i | 1 \leq i \leq m\}$.*

Although the second way of computing minimal explanations seems to introduce an unnecessary detour, it may be useful when the computation of $CNF((\sigma_1 \wedge \ldots \wedge \sigma_k) \supset \epsilon)\}$ is quite expensive, for example when Σ is a logic program. In fact, computing $CNF(Th(M_1) \vee \ldots \vee Th(M_r))$ can be more direct, as one deals only with minimal models.

A method to get rid of all explanations inconsistent with Σ can easily be obtained. We describe it as a refinement of Corollary 1: the set of minimal explanations for ϵ in the context of $\Sigma = \{\sigma_1, \ldots, \sigma_k\}$ is $\{Th(M) | M$ is a minimal three-valued model of $CNF((\sigma_1 \wedge \ldots \wedge \sigma_k) \supset \epsilon)$ and M is not a minimal three-valued model of $CNF(\neg(\sigma_1 \wedge \ldots \wedge \sigma_k))\}$.

An immediate corollary of the above results is the following, that relates prime implicants and prime implicates of the same formula.

Corollary 3 *If $\phi_1, \ldots, \phi_n$ are all the prime implicants of a ground formula ψ and $\xi_1 \ldots \xi_m$ are all the prime implicates of ψ, then $\xi_1 \wedge \ldots \wedge \xi_m = CNF(\phi_1 \vee \ldots \vee \phi_n)$.*

Here follows an example showing that two-valued minimal models cannot be used instead of three-valued ones. Let $\Sigma = \{q \supset p, \neg r \wedge \neg p \supset q, \neg q \wedge p \supset s\}$ and q the observation to be explained. The minimal three-valued models for $\Sigma \cup \{\neg q\}$ are $M_1 = \{r, \neg q, \neg p\}$, $M_2 = \{r, \neg q, s\}$ and $M_3 = \{p, \neg q, s\}$. $CNF(Th(M_1) \vee Th(M_2) \vee Th(M_3)) = (r \vee p) \wedge (r \vee s) \wedge \neg q \wedge (\neg p \vee s)$. Thus (Corollary 2) the explanations for q are, besides the trivial one, $\phi_1 = \neg r \wedge \neg p$, $\phi_2 = \neg r \wedge \neg s$ and $\phi_3 = p \wedge \neg s$. The minimal two-valued models of $\Sigma \cup \{\neg q\}$ are $M_1' = \{r, \neg q, \neg p, \neg s\}$ and $M_2' = \{r, \neg q, s, \neg r\}$ (in two-valued models negative literals are normally omitted). $CNF(Th(M_1') \vee Th(M_2')) = (r \vee p) \wedge (r \vee s) \wedge \neg q \wedge (\neg p \vee s) \wedge (\neg p \vee \neg r) \wedge (\neg s \vee p) \wedge (\neg s \vee \neg r)$. Thus, besides the explanations obtained by means of three-valued models, also the following conjunctions are generated: $(p \wedge r)$, $(\neg p \wedge s)$, $(s \wedge r)$, that are not explanations for q in Σ.

4 The simplest first order case

The results obtained so far for ground abduction can be immediately lifted to the case where the theory Σ contains only purely universal sentences and ϵ is a purely existential sentence. We state here the method in terms of prime implicates, because this does not require the introduction of any new concept. Needless to say, the dual method in terms of prime implicants is readily obtainable.

Let Γ be a set of purely universal formulae. For any $k = 0, 1, 2, \ldots$, we define the extension $\mathcal{L}_k^-$ of the language $\mathcal{L}^-$, obtained by adding k new constants $m_1, \ldots, m_k$. Γ_k^* is the set of all the ground instances of Γ in the Herbrand universe of $\mathcal{L}_k^-$. For any sentence ϕ, let $deH(\phi)$ be obtained from ϕ by replacing each constant m_i (not in $\mathcal{L}^-$) by a new universally quantified variable. Thus $deH(\phi)$ has the form $\forall y_1 \ldots \forall y_r (L_1 \vee \ldots \vee L_m)$. The prime implicates of Γ are among the elements in the set

$$\bigcup_{k=0}^{\infty} \{deH(\phi) \mid \phi \text{ is a prime implicate of } \Gamma_k^*\}.$$

In fact, if Γ is purely universal and $sk(\Gamma)$ is the skolemization of the formulae in Γ, and $sk(\phi) = \phi[m_1, ..., m_k]$ is the skolemization of ϕ, obtained by use of the herbrand constants $m_1, ..., m_k$:

$$
\begin{aligned}
\Gamma \models \phi \text{ iff } & \Gamma \cup \{\neg\phi\} \text{ is unsatisfiable} \\
\text{iff } & sk(\Gamma \cup \{\neg\phi\}) \text{ is unsatisfiable} \\
\text{iff } & \Gamma \cup \{sk(\neg\phi)\} \text{ is unsatisfiable} \\
\text{iff } & \Gamma \cup \{\neg\phi[m_1, ..., m_k]\} \text{ is unsatisfiable} \\
\text{iff } & \Gamma \cup \{\neg\phi[m_1, ..., m_k]\} \text{ has no Herbrand models} \\
\text{iff } & \Gamma^*_k \cup \{\neg\phi[m_1, ..., m_k]\} \text{ has no Herbrand models} \\
\text{iff } & \Gamma^*_k \models \phi[m_1, ..., m_k].
\end{aligned}
$$

Finally, $\phi = deH(\phi[m_1, ..., m_k])$ (it is clear that any *prime* implicate of a purely universal set of formulae is purely universal, too).

If Σ is purely universal and ϵ purely existential, then the minimal explanations for ϵ in the theory Σ are among the negations of the prime implicates of $\Sigma \cup \{\neg\epsilon\}$, thus they are existential conjunctions of literals.

The method described here is a simplified form of reverse skolemization [3]. A much less explosive method for lifting the propositional results to first order is, of course, by use of a unification mechanism. The general approach to first order abduction will be developed in a forthcoming paper, where neither the language nor the form of the theory/observation are submitted to restrictions.

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